Chleboun, Jan

On a reliable solution of a quasilinear elliptic equation with uncertain coefficients: Sensitivity analysis and numerical examples. (English) Zbl 1002.35041

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 44, No. 3, 375-388 (2001).

From the introduction: The aim of the paper is to add sensitivity analysis and numerical tests to the existence and convergence results published in [*I. Hlaváček*, Reliable solution of a quasilinear nonpotential elliptic problem of a nonmonotone type with respect to the uncertainty in coefficients, J. Math. Anal. Appl. 212, 452-466 (1997; Zbl 0919.35047)]. The isotropic material case is studied in [*J. Chleboun*, Reliable solution for 1D quasilinear elliptic equations with uncertain coefficients, J. Math. Anal. Appl. 234, No. 2, 514-528 (1999; Zbl 0944.35027)]. By way of contrast, anisotropic medium is considered in this paper. The mathematical problem examined in the paper has a clear physical meaning. We consider a steady-state heat flow in an anisotropic body. The temperature distribution is modeled by a quasilinear elliptic equation with uncertain coefficients of heat conductivity. These are temperature dependent and belong to an admissible set derived from measurements, for example. We choose a small test subdomain G and look for the difference between the highest and the lowest mean temperature we can get on G taking into account admissible conductivities. Since the body is anisotropic, the Kirchhoff transformation cannot be applied to get rid of the nonlinearity in the state equation. Also, cost functional gradient computation is more complex than in the case of an isotropic material.

MSC:

35J60 Nonlinear elliptic equations
35A15 Variational methods applied to PDEs

Keywords:

approximation; existence; convergence results; cost functional; steady-state heat flow; isotropic material case; anisotropic medium

Software:

NAG Foundation Toolbox; Matlab; Partial Differential Equation Toolbox

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