Suslin, Andrei; Voevodsky, Vladimir

Bloch-Kato conjecture and motivic cohomology with finite coefficients. (English)

Zbl 1005.19001
Gordon, B. Brent (ed.) et al., The arithmetic and geometry of algebraic cycles. Proceedings of the NATO Advanced Study Institute, Banff, Canada, June 7-19, 1998. Vol. 1. Dordrecht: Kluwer Academic Publishers. NATO ASI Ser., Ser. C, Math. Phys. Sci. 548, 117-189 (2000).

In the paper the authors prove that the Beilinson-Lichtenbaum conjecture is equivalent to the Bloch-Kato conjecture. The conjecture of Beilinson and Lichtenbaum describes the motivic cohomology of smooth varieties with finite coefficients and for weight n it states that the natural map

$$Z/l(n) \to \tau_{\leq n} R(\pi_0)_*(\mu_l^{\otimes n})$$

is a quasi-isomorphism. The Bloch-Kato conjecture relates Milnor K-theory to Galois cohomology and says that for a field E the norm residue homomorphism $K_n^M(E)/l \to H^n_{\text{ét}}(E, \mu_l^{\otimes n})$ is surjective.

The first five sections of the paper are devoted to review the definition of motivic cohomology and its main properties and may serve as the introduction to the subject. The proof of the main theorem is given in sections 6-10. The proof requires resolution of singularities over the base field and therefore applies in the characteristic zero case. In section 11 the authors show that the Beilinson-Lichtenbaum conjecture can be reduced to show that certain Bockstein operations in étale cohomology are zero.

For the entire collection see [Zbl 0933.00032].

Reviewer: Piotr Krasoń (Szczecin)

MSC:

19E15	Algebraic cycles and motivic cohomology (K-theoretic aspects)
14F42	Motivic cohomology; motivic homotopy theory
14C35	Applications of methods of algebraic K-theory in algebraic geometry

Keywords:

motivic cohomology; Beilinson-Lichtenbaum conjecture; Bloch-Kato conjecture; resolution of singularities; étale cohomology