Martel, Yvan; Merle, Frank

Stability of blow-up profile and lower bounds for blow-up rate for the critical generalized KdV equation. (English) Zbl 1005.35081

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The generalized KdV equation $u_t + (u_{xx} + u^p)_x = 0$ ($0 \leq t, -\infty < x < \infty$, $p \geq 2$ is an integer), $u(0,x) = u_0(x) \in H^1(\mathbb{R})$, has global and bounded in time solutions $u(t) \in H^1(\mathbb{R})$ if $p < 5$. The article is devoted to the critical case $p = 5$.

Let $Q = 3^{1/4}/\cosh^{1/2} 2x$ be the critical state (solution of $Q_{xx} + Q^5 = Q$) and $E(u) = \frac{1}{2}\int u_x^2 - \frac{1}{6}\int u^6$ be the energy. Assuming $|u_0|_{L^2} < |Q|_{L^2}$, the solution $u(t) \in H^1(\mathbb{R})$ is global and uniformly bounded. On the other hand, if $|u_0|_{L^2} > |Q|_{L^2}$, then there exists $\alpha_0 > 0$ such that for all $u_0 \in H^1(\mathbb{R})$ with $E(u_0) < 0$ and $\int u_0^2 \leq \int Q^2 + \alpha_0$, there are blow-up solutions $|u(t)|_{H^1} \to \infty$ as $t \to T$ (where $T \leq \infty$). In the latter case, the article involves two qualitative results.

Theorem on the stability of the blow-up profile: There exist $\lambda(t) \geq 0$ and $x(t) \in \mathbb{R}$ such that the function $\lambda^{1/2}(t)u(t, \lambda(t)x - x(t))$ weakly converges in $H^1(\mathbb{R})$ either to Q or to $-Q$ as $t \to T$.

Theorem on the blow-up rate: $\lim (T-t)^{1/3}|u_x(t)|_{L^2} = \infty$.

Reviewer: Jan Chrastina (Brno)

MSC:

35Q53 KdV equations (Korteweg-de Vries equations)
37K45 Stability problems for infinite-dimensional Hamiltonian and Lagrangian systems

Keywords:

KdV equation; blow-up solutions

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