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Three-dimensional metrics as deformations of a constant curvature metric. (English)

Zbl 1008.53039

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The following result is proved: Any three-dimensional Riemannian metric g may be locally obtained from a constant curvature metric h by a deformation of the form $g = \sigma h + \varepsilon s \otimes s$, where σ and s are respectively a scalar function and a differential 1-form; the sign $\varepsilon = \pm 1$ and a relationship $\Psi(\sigma, |s|)$ between the scalar σ and the Riemannian norm $|s|$ may be arbitrarily prescribed. Some examples are also presented (Schwarzschild space and Kerr space).

Reviewer: Marian Munteanu (Iasi)

MSC:

53C21	Methods of global Riemannian geometry, including PDE methods; curvature restrictions
58H15	Deformations of general structures on manifolds
83C99	General relativity

Keywords:

constant curvature metric; flat deformation; Cauchy problem

Full Text: DOI

References:

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[2] Eisenhart, L. P., (1960). A Treatise on the Differential Geometry of Curves and Surfaces, Dover Publications, Inc. New York, p 93. · Zbl 0090.37803

[3] In spite of its interest, the known results on diagonalization of three-dimensional metrics do not belong to this type: in all of them, in addition to the f D 3 scalars, an orthogonal tetrad is also involved with the (more o less implicit) flat metric. See Cartan, E., Les syst'emes differentials ext'erieurs et leurs applications g'eom'etriques, Hermann, Paris, 1945 and 1971 for an analytic proof on the existence of orthogonal coordinates; Deturck, M., Duke Math. Jour., 51, p 243–60, (1984) for a C1 proof;Walberer, P., Abhandl. Math. Sem. Univ. Hamburg, 10, p 169–79, 1934 for the decomposition in a given orthogonal frame; Bel, L., Gen. Rel. Grav., 28, p 1139–50, (1996) for decompositions in principal frames; Tanno, S., J. Differential Geometry, 11, p 467–74, (1976) for other particular decompositions.

[4] Coll, B., (1999). A Universal Law of Gravitational Deformation for General Relativity, in Proceedings of the Spanish Relativity Meeting in honour of the 65th Birthday of Lluis Bel "Gravitation and Relativity in General" ed. by Martin, J., Ruiz, E., Atrio, F., Molina, A., World Scientific.

[5] See for example O'Neill, B., Elementary Differential Geometry, Academic Press, 1966.

[6] Lichnerowicz, A., (194

[7] We shall use the definition of the curvature tensor in terms of a general basis given in: Choquet-Bruhat, Y., Dewitt-Morette, C., and Dillard-Bleick, M., Analysis, Manifolds and Physics, p. 306, revised edition, North-Holland (Amsterdam, 1987). Nevertheless, we shall write the first pair of indices in the positions Ri jkl as in Eisenhart, L. P., Riemannian Geometry, eq. (8.3), Princeton University Press (Princeton, N.J. 1997). · Zbl 0385.58001

[8] Eisenhart, L. P., Op. cit., eq. (28.12).

[9] Eisenhart, L. P., Op. cit., p. 83.

[10] Eisenhart, L. P., Op. cit., eq. (26.2) Princeton University Press (Princeton, N.J. 1997).

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