Let K be a field of characteristic zero complete with respect to some discrete valuation. If K_0 denotes the field of fractions of the associated ring of Witt vectors, then K/K_0 is a finite totally ramified field extension. A p-adic representation of the Galois group $G_K = \mathrm{Gal}(\overline{K}/K)$ is given by a $\mathbb{Q}_p$-vector space of finite dimension, together with a continuous linear action of G_K on it. Among those representations are the so-called semi-stable representations, which are related to étale cohomology theory in algebraic geometry, and for any semi-stable p-adic representation, there is an associated algebraic object, namely its so-called (φ, N)-module D. The correspondence that associates to a representation V its filtered (φ, N)-module D defines a tensor equivalence between the category of semi-stable p-adic representations of G_K and a full subcategory of the category of (φ, N)-modules, which is the so-called category of admissible filtered (φ, N)-modules. The second author of the present paper, *J.-M. Fontaine*, has studied this functor in some of his earlier works between 1979 and 1994. However, up to now, the entire theory did not appear to be complete, as no explicit description of the category of admissible filtered (φ, N)-modules had been achieved so far.

In the paper under review, the authors prove a fact that had been conjectured in the past. Namely, one can define the category of weakly admissible filtered (φ, N)-modules, which seems to be easier to come to grips with, and then the conjecture states that weak admissibility is indeed the same as ordinary admissibility.

The fact that this equivalence holds is a consequence from theorem A proved in the present paper. This principal theorem, which transpires a wealth of important corollaries, with a view to semi-stable p-adic representations of the Galois group G_K, is proved by a fine and involved analysis of (φ, N)-modules using the structure of the extension K/K_0 and previous results of both authors.

The importance of the authors' main result, theorem A, and its consequences becomes even more evident in the light of the recent fundamental results of *J.-P. Wintenberger* on p-adic representations and their relations to p-adic Hodge theory, which are based on the authors' now fully established, crucial equivalence theorem. Wintenberger's results are stated (and referred to) in the paper under review, which helps the reader to grasp the bearing of theorem A.

Also, the utilized earlier results of J.-M. Fontaine's, which play a prominent rôle in the course of the proof of the central result of the present paper, are carefully restated and reviewed, which makes the entire exposition largely self-contained and easily readable.

Reviewer: Werner Kleinert (Berlin)

MSC:

14G20	Local ground fields in algebraic geometry
11S31	Class field theory; p-adic formal groups
14F30	p-adic cohomology, crystalline cohomology
11R32	Galois theory

Cited in **9** Reviews
Cited in **58** Documents

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Tannaka categories; crystalline cohomology; p-adic étale cohomology; p-adic representation of the Galois group; category of admissible filtred modules; Witt vectors

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