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Singularly perturbed boundary value problems for elliptic equation with a curve of turning point. (English) [Zbl 1010.35030]

The following singularly perturbed problem is studied:

\[ L_\varepsilon u = \varepsilon L u + f(x, y) \frac{\partial u}{\partial y} + g(x, y) u = 0, \ (x, y) \in \Omega \]

with

\[ u(x, y) = \varphi_1(x), \ a \leq x \leq b, \ y \in \partial \Omega_1 \]

and

\[ u(x, y) = \varphi_2(x), \ a \leq x \leq b, \ y \in \partial \Omega_2, \]

where \( \varepsilon > 0 \) and

\[ L = a_{1,1}(x, y) \frac{\partial^2}{\partial x^2} + 2a_{1,2}(x, y) \frac{\partial^2}{\partial x \partial y} + a_{2,2}(x, y) \frac{\partial^2}{\partial y^2}. \]

We assume an ellipticity condition for \( L \): There exists \( \lambda > 0 \) such that, for all \( \xi = (\xi_1, \xi_2) \in \mathbb{R}^2 \),

\[ a_{1,1}\xi_1^2 + 2a_{1,2}\xi_1\xi_2 + a_{2,2}\xi_2^2 \geq \lambda (\xi_1^2 + \xi_2^2). \]

Moreover, \( \Omega = \Omega_1 \cup \Omega_2 \) is a smooth bounded domain in \( \mathbb{R}^2 \).

It is proved that the solution \( u(x, y, \varepsilon) \) of this problem has the following expansion:

\[ u(x, y, \varepsilon) = \varphi_1(x) \exp \left( - \frac{1}{\varepsilon} \int_{y_1(x)}^{y} \frac{f(x, y')}{{a_{2,2}(x, y')}} \, dy' \right) + \varphi_2(x) \exp \left( - \frac{1}{\varepsilon} \int_{y_2(x)}^{y} \frac{f(x, y')}{{a_{2,2}(x, y')}} \, dy' \right) + O(\varepsilon^{1/2}) \]

when \( \varepsilon \to 0 \).

Reviewer: Emmanuel Russ (Marseille)

MSC:
35J25 Boundary value problems for second-order elliptic equations
35B25 Singular perturbations in context of PDEs
35C20 Asymptotic expansions of solutions to PDEs

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singular perturbation; turning point; elliptic equation

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References:


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