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Integrable nonlinear evolution equations on the half-line. (English) Zbl 1010.35089

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Summary: A rigorous methodology for the analysis of initial-boundary value problems on the half-line, $0 < x < \infty$, $t > 0$, is applied to the nonlinear Schrödinger (NLS), to the sine-Gordon (sG) in laboratory coordinates, and to the Korteweg-deVries (KdV) equation with dominant surface tension. Decaying initial conditions as well as a smooth subset of the boundary values $\{\partial_x^l q(0,t) = g_l(t)\}_0^{n-1}$ are given, where $n = 2$ for the NLS and the sG and $n = 3$ for the KdV. For the NLS and the KdV equations, the initial condition $q(x,0) = q_0(x)$ as well as one and two boundary conditions are given respectively; for the sG equation the initial conditions $q(x,0) = q_0(x), q_t(x,0) = q_1(x)$, as well as one boundary condition are given. The construction of the solution $q(x,t)$ of any of these problems involves two separate steps:

(a) Given decaying initial conditions define the spectral (scattering) functions $a(k), b(k)$. Associated with the smooth functions $\{g_l(t)\}_0^{n-1}$, define the spectral functions $A(k), B(k)$. Define the function $q(x,t)$ in terms of the solution of a matrix Riemann-Hilbert problem formulated in the complex k-plane and uniquely defined in terms of the spectral functions $\{a(k), b(k), A(k), B(k)\}$. Under the assumption that there exist functions $\{g_l(t)\}_0^{n-1}$ such that the spectral functions satisfy a certain global algebraic relation, prove that the function $q(x,t)$ is defined for all $0 < x < \infty$, $t > 0$, it satisfies the given nonlinear PDE, and furthermore that $q(x,0) = q_0(x), \{\partial_x^l q(0,t) = g_l(t)\}_0^{n-1}$.

(b) Given a subset of the functions $\{g_l(t)\}_0^{n-1}$ as boundary conditions, prove that the above algebraic relation characterizes the unknown part of this set. In general this involves the solution of a nonlinear Volterra integral equation which is shown to have a global solution. For a particular class of boundary conditions, called linearizable, this nonlinear equation can be bypassed and $\{A(k), B(k)\}$ can be constructed using only the algebraic manipulation of the global relation. For the NLS, the sG, and the KdV, the following particular linearizable cases are solved: $q_x(0,t) - \chi q(0,t) = 0$, $q(0,t) = \chi$, $\{q(0,t) = \chi, q_{xx}(0,t) = \chi + 3\chi^2\}$, respectively, where χ is a real constant.

MSC:

35Q53	KdV equations (Korteweg-de Vries equations)
35Q55	NLS equations (nonlinear Schrödinger equations)
35Q15	Riemann-Hilbert problems in context of PDEs
37K15	Inverse spectral and scattering methods for infinite-dimensional Hamiltonian and Lagrangian systems

Keywords:

linearizable boundary conditions; nonlinear Schrödinger; sine-Gordon; Koreteweg-deVries; decaying initial conditions; spectral functions

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