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Nonlocal Cauchy problems for first-order multivalued differential equations. (English)

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Electron. J. Differ. Equ. 2002, Paper No. 47, 9 p. (2002).

It is investigated the existence of solutions for the Cauchy problem

$$\dot{x} \in F(t, x(t)), \quad x(0) + \sum_{k=1}^{m} a_k x(t_k) = 0, \quad t \in (0, T]. \tag{1}$$

Here, $F : J \times \mathbb{R} \to 2^{\mathbb{R}}$ is a set-valued map, $J = [0, T], 0 < t_1 < t_2 \ldots t_m < 1$. The set of all bounded closed convex and nonempty subsets of $\mathbb{R}$ is denoted by $bcc(\mathbb{R})$. The following conditions are assumed:

(H_0) $a_k \neq 0$ for each $k = 1, 2, \ldots, m$, and $\sum_{k=1}^{m} a_k + 1 \neq 0$.

(H_1) $F : J \times \mathbb{R} \to bcc(\mathbb{R}), (t, x) \to F(t, x)$ is measurable in t for each $x \in \mathbb{R}$ and upper semicontinuous with respect to $x \in \mathbb{R}$ for a.e. $t \in J$. (H_2) $|F(t, x)| \leq \psi(|x|)$ for a.e. $t \in J$ and all $x \in \mathbb{R}$, where $\psi : [0, \infty) \to (0, \infty)$ is continuous nondecreasing and such that $\limsup_{\rho \to \infty} \psi(\rho)/\rho = 0$.

Under these conditions the following theorem holds: If the assumptions $(H_0), (H_1), (H_2)$ are satisfied, then the initial value problem (1) has least one solution.

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MSC:

34A12 Initial value problems, existence, uniqueness, continuous dependence and continuation of solutions to ordinary differential equations
34A60 Ordinary differential inclusions
34G20 Nonlinear differential equations in abstract spaces

Keywords:

Cauchy problems; multivalued differential equations; nonlocal condition; topological transversality theorem

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