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The mesa-limit of the porous-medium equation and the Hele-Shaw problem. (English)

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Differ. Integral Equ. 15, No. 2, 129-146 (2002).

Summary: We are interested in the limit, as $m \to \infty$, of the solution u_m of the porous-medium equation $u_t = \Delta u^m$ in a bounded domain Ω with Neumann boundary condition, $\frac{\partial u^m}{\partial n} = g$ on $\partial\Omega$, and initial datum $u(0) = u_0 \geq 0$. It is well known by now that this kind of limits turns out to be singular. In the case $g \equiv 0$, it was proved that there exists an initial boundary layer $\underline{u}_0$, the so-called mesa, and $u_m(t) \to \underline{u}_0$, in $L^1(\Omega)$, for any $t > 0$, as $m \to \infty$. In this work, we generalize this result to the case of arbitrary $g \in L^2(\partial\Omega)$, we prove that the initial boundary layer is still $\underline{u}_0$ and in general (even in the regular case) the limit function is not a solution of a Hele-Shaw problem. There exists a time interval I where the limit of u_m, as $m \to \infty$, is the unique solution of a Hele-Shaw problem and elsewhere, u_m converges to the constant function $\frac{1}{|\Omega|}(\int_\Omega u_0 + t\int_{\partial\Omega} g)$.

MSC:

35K60	Nonlinear initial, boundary and initial-boundary value problems for linear parabolic equations
35K65	Degenerate parabolic equations
35A05	General existence and uniqueness theorems (PDE) (MSC2000)
35B40	Asymptotic behavior of solutions to PDEs

Keywords:

Neumann boundary condition; initial boundary layers