

Romanowska, Anna B.; Smith, Jonathan D. H.

Modes. (English) Zbl 1012.08001

Singapore: World Scientific. xii, 623 p. (2002).

Let \mathcal{A} be an algebra defined on the set A with operations $\{f_i\}$. If all f_i satisfy $f_i(a, \dots, a) = a$ for all $a \in A$, then \mathcal{A} is said to be idempotent. If all f_i are homomorphisms, then \mathcal{A} is said to be entropic. If both conditions hold, then \mathcal{A} is a mode. A simple example of a mode is a Steiner quasigroup (a groupoid satisfying the laws $x^2 = x$, $xy = yx$ and $x(xy) = y$) with the additional law $(xy)(zt) = (xz)(yt)$. (Those knowledgeable about Steiner triple systems will recognise these as the Steiner quasigroups arising from affine triple systems.) As its title suggests, this monograph gives an extensive survey of the theory of modes. The first four chapters give an introduction to classes of algebras, then chapters five to eight are devoted to the theory of modes. Chapter nine shows how modes can be applied in the theory of hierarchical statistical mechanics. The final chapter gives an interesting survey of recent developments and open problems. At the end of each of chapters four to eight, useful “historical” notes are provided giving at least the most essential references for that particular chapter, and there is a bibliography covering more than 40 pages. There are excellent exercises in every chapter except the last, but some hints for solutions would be most welcome. Perhaps these could be provided on the second author’s webpage. I found few misprints, and those there are should cause no confusion (even the “involuntary” isomorphism on page 299). However, the refusal (or inability) of the authors to draw slanting arrows leads to some very strange Hasse diagrams. I found the diagrams for N_5 and M_3 on page 111 particularly startling! The frequent use of double-underlining makes the presentation rather ugly; with the variety of fonts now available in most typesetting packages one feels this could have been avoided. I also found the chapter on statistical mechanics very hard to follow, and wonder if a simpler application could not have been found, and perhaps one of more interest to the average universal algebraist, the likely reader of this book! One of my main worries about this work is for whom it is intended. The provision of exercises suggest it is meant as a graduate text, but the amount of material in it would be difficult to cover in a year course, let alone a semester, although the chapter on binary modes, with the necessary preliminaries would indeed make a nice course.

Reviewer: [Sheila Oates-Williams \(Beerwah\)](#)

MSC:

- [08-02](#) Research exposition (monographs, survey articles) pertaining to general algebraic systems
- [08A05](#) Structure theory of algebraic structures
- [08B05](#) Equational logic, Mal'tsev conditions
- [08C15](#) Quasivarieties
- [18B99](#) Special categories

Cited in 5 Reviews Cited in 32 Documents

Keywords:

[idempotent entropic algebras](#)