Koszul rings are an important class of graded rings with applications in algebraic topology, algebraic geometry, noncommutative algebraic geometry and in the theory of quantum groups. The authors develop a duality theory that unifies Koszul duality and duality for cotilting modules. The paper is divided into four chapters: Main results and examples (Chapter I), Proof of main results (II), Generalized Koszul algebras (III) and Further results and questions (IV).

Let $\Gamma = \bigoplus_{i \geq 0} \Gamma_i$ be a positively $\mathbb{Z}$-graded algebra generated in degrees 0 and 1 and let $T$ be a cotilting $\Gamma_0$ module as introduced by T. Wakamatsu [J. Algebra 134, No. 2, 298-325 (1990; Zbl 0726.16009)]. The authors define (strongly) Koszul algebras with respect to $T$ and the category of $T$-Koszul modules $K_T(\Gamma)$ and show, under certain conditions, the existence of a duality $E : K_T(\Gamma) \to K_T(\Lambda)$ defined by $E(M) = \bigoplus_{n \geq 0} \text{Ext}^n_{\Gamma}(M, T)$, where $\Lambda = \bigoplus_{n \geq 0} \text{Ext}^n_{\Gamma}(T, T)$ is the Yoneda algebra. It is shown that Koszul algebras are quadratic and several examples, different from the classical ones, are presented. In Chapter III, after studying deeply the category of Koszul modules over Koszul algebras, a different approach is used to study generalized Koszul algebras and this notion is compared with their first definition and the classical cases. It is important to remark that it is shown that the conditions used in the definition of Koszul algebras, are almost necessary to have a good duality theory. In last chapter, it is shown that if $\Gamma = \Gamma_0 + \Gamma_1 + 0 + 0 + \cdots$ is a $T$-Koszul algebra with respect to a cotilting $\Gamma_0$-module $T$, then $\Gamma$ is strongly $T$-Koszul and other sufficient conditions for this implication are obtained. The book finishes with several open problems and some partial results on them.

This book will be very important in the development of the Koszul duality theory and representation theory.

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MSC:

16S37 Quadratic and Koszul algebras
16D90 Module categories in associative algebras
16W50 Graded rings and modules (associative rings and algebras)
18E05 Preadditive, additive categories
16E05 Syzygies, resolutions, complexes in associative algebras
16E30 Homological functors on modules (Tor, Ext, etc.) in associative algebras

Keywords:
Koszul algebras; dualities; cotilting modules; categories of Koszul modules; Yoneda algebras; quadratic algebras

Full Text: DOI