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On the proalgebraic completion of a finitely generated group. (English) Zbl 1012.20039

Cleary, Sean (ed.) et al., Combinatorial and geometric group theory. Proceedings of the AMS special session on combinatorial group theory, New York, NY, USA, November 4-5, 2000 and the AMS special session on computational group theory, Hoboken, NJ, USA, April 28-29, 2001. Providence, RI: American Mathematical Society (AMS). Contemp. Math. 296, 171-181 (2002).

Given a finitely generated discrete group Γ, the proalgebraic completion $A(\Gamma)$ of Γ is a proalgebraic group that is universal with respect to finite dimensional representations of Γ. In recent work of *H. Bass, A. Lubotzky, S. Mozes* and the author [Geom. Dedicata 95, 19-58 (2002; Zbl 1059.20036)], the structure of $A(\Gamma)$ was investigated. Building on those results, the author here obtains a structure theorem about the coordinate algebra $k[A(\Gamma)]$ (with k being an algebraically closed field of characteristic zero). Specifically, $k[A(\Gamma)]$ is identified with the tensor product $k[A^0(\Gamma)]\otimes k[\widehat{\Gamma}]$ where $A^0(\Gamma)$ is the identity component of $A(\Gamma)$ and $\widehat{\Gamma}$ is the profinite completion of Γ. This result is a special case of a more general theorem proved for a proaffine proalgebraic group G: $k[G]\simeq k[G^0]\otimes k[G/G^0]$, where G^0 denotes the identity component.

The theorem allows one to identify induction from G^0-modules to G-modules as tensoring with $k[G/G^0]$. The author goes on to investigate induction in the context of $A(\Gamma)$. The category of $A^0(\Gamma)$-modules is identified with the category of so-called 'virtual' Γ-modules. Hence one obtains information on induction from virtual Γ-modules to Γ-modules.

For the entire collection see [Zbl 0990.00044].

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20G05 Representation theory for linear algebraic groups

20E18 Limits, profinite groups

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