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**Critical exponents for Brownian motion and random walks. (Exposants critiques pour le mouvement brownien et les marches aléatoires.)** (French) Zbl 1012.60072

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The paper has, corresponding to its title, two parts, the first follows Lawler and Werner, the second Kenyon, mainly. Proofs are sketched in Part 1. If $B_t$ is a process, $B[0,a]$ denotes $\{B_t; t \in [0,a]\}$ and $T_R(B)$ the first visit in $(|\cdot| = R)$ of $B$ (the considered processes have $\mathbb{C}$ as state space). If $A \subset \mathbb{C}$, consider $Z_R(A) = P(W[0,T_R(W)] \cap A = \emptyset)$, where $W$ is a Brownian motion with $W_0$ uniformly distributed on $(|\cdot| = 1)$. For $\lambda \geq 0$ one defines

$$\zeta(1,\lambda) = -\lim_{R\to\infty} \log E(Z_R(B[0,T_R(B)])^\lambda),$$

where $B$ is a Brownian motion as $W$. In an analogous manner one defines "the exponents" $\zeta(n_1,\lambda_1,\dots,n_k,\lambda_k)$, $n_i \geq 1$, $\lambda_i \geq 0$, using a family of $n_1+\dots+n_k$ independent Brownians, divided in $k$ groups, containing $n_1,\dots,n_k$ of them respectively and multiplying the $Z^\lambda$'s. In the same way one defines $\xi(\lambda_0,n_1,\lambda_1,\dots,n_k,\lambda_k)$ requiring in $Z_R$ also that $W[0,T_R(W)] \subset (\mathrm{Im} > 0)$, $\lambda_0$ appearing when $\subset (\mathrm{Im} > 0)$ is the single requirement. The $\xi$'s satisfy $\xi(\lambda_0,n_1,\dots,\lambda_k) = \xi(\lambda_0,n_1,\dots,n_j,\xi(\lambda_j,n_{j+1},\dots,\lambda_k))$, while the $\zeta$'s satisfy an analogous relation, with only the first two $\xi$'s replaced by $\zeta$'s (and without $\lambda_0$).

Defining also $\xi(\alpha,\alpha')$ by $\xi(0,1,\xi(\alpha,\alpha')) = \xi(\alpha,1,\alpha')$, $\xi(\alpha_1,\dots,\alpha_k) = \xi(\alpha_1,\xi(\alpha_2,\dots,\alpha_k))$, $\zeta(\alpha_1,\dots,\alpha_k) = \zeta(1,\xi(\alpha_1,\dots,\alpha_{j-1},\beta_j,\alpha_{j+1},\dots,\alpha_k))$, $\alpha_j \geq 1$, $\alpha_j = \xi(1,\beta_j)$, the author states as main theorem the existence of a homeomorphism $U : [0,\infty) \to [0,\infty)$ such that

$$\xi(\alpha_1,\dots,\alpha_k) = U^{-1}(U(\alpha_1)+\dots+U(\alpha_k))$$

and that $\zeta(\alpha_1,\dots,\alpha_k)$ is a function of $\xi(\alpha_1,\dots,\alpha_k)$. As applications of these concepts we mention the study of the Hausdorff dimension of the frontier of the component of $\infty$ in $B([0,T_R(B)])^c$ and of the set of $z$'s which disconnect $B[0,1]$. Then more recent results are mentioned: results allowing explicit calculations of $\xi$,$\zeta$,$U$ and definition of exponents for finite families of measures on the set of all "oriented bridges" between two arcs of $(|\cdot| = 1)$ and their relations with the $\xi$'s.

The exact title of the second part is: Random walks with deleted loops and domino pavings. The first concept means replacing $(\gamma_n)_{0\leq n\leq N}$ by $(\delta_n)_{0\leq n\leq N'}$ where $k = \min\{r; \gamma_r \in \{\gamma_0,\dots,\gamma_{r-1}\}\}$, $\gamma_k = \gamma_i$, $i < k$, $\delta_n = \gamma_n$ for $n \leq i$ and $\delta_{i+n} = \gamma_{k+n}$ for $n \geq 1$ and repeating this procedure until possible. For the usual random walks on $\mathbb{N}\times\mathbb{Z}$ and on $\mathbb{Z}^2$ it was shown to be possible to define a corresponding random walk with deleted loops (RWDL) and the probability that $re^{i\theta}$ is visited by that on $\mathbb{N}\times\mathbb{Z}$ starting from 0 is $r^{-(3/4)(1+o(1))}((\cos\theta)^{1/4}+o(1))$ for $r\to\infty$. It is shown that the RWDL starting from 0, on $\mathbb{Z}^d$, $d \leq 4$, and on $\mathbb{N}\times\mathbb{Z}$, has as law the limit of the uniform distribution on the set of all trees which are subgraphs of $[-n,n]^d \cap \mathbb{Z}^d$, $[-n,n]^2 \cap (\mathbb{N}\times\mathbb{Z})$ containing all its vertices.

Consider now the "chess paving" of $\mathbb{R}^2$ composed of unit squares, the set of their centers being $\mathbb{Z}^2$ and such that the one centered in $(0,0)$ is white. Let $W_0$ be the set of all white squares with centers $(2m,2n)$, $W_1$ the set of all other white ones, $B_0$ the set of all blacks with centers $(2m+1,2n+1)$, $B_1$ the set of all other blacks. A temperlien polyomino is a "union $P$ of squares of this paving, having as frontier a simple curve" (property (u)), such that all corners correspond to squares in $B_1$ and from which a black square on the frontier is deleted (named the basic square of $P$). If $M(P)$ is the set of its squares, one defines the matrix $K(v,w)$, $v,w \in M$, by $K(v,w) = 1,i,-1,-i$ if $w$ is the right, upper, left, lower neighbor of $v$, respectively, and 0 otherwise. Let $C(P) = K^{-1}$. Then $|C(P)(v,w)|$ is the probability that, choosing at random a decomposition of $P$ into "dominoes" (i.e. unions of two adjacent unit squares), the domino $vw$ figures in it. Let $U$ have property (u) and be approximated ($\varepsilon\to 0$) by $\varepsilon P_\varepsilon$ where $P_\varepsilon$ are temperlien polyominoes (corner by corner and with basic points convergent to a $d_0$) and let $\varepsilon v_\varepsilon \to v$, $\varepsilon w_\varepsilon \to w$, $v,w \in U$, $v_\varepsilon, w_\varepsilon \in M(P_\varepsilon)$. Then $\lim_{\varepsilon\to 0} C(P_\varepsilon)(v_\varepsilon,w_\varepsilon)$ exists and equals $\mathrm{Re}F_0(v,w)$, $i\mathrm{Im}F_0(v,w)$, $\mathrm{Re}F_1(v,w)$, $i\mathrm{Im}F_1(v,w)$ if, for all $\varepsilon$, "$v_\varepsilon \in W_0$, $w_\varepsilon \in B_0$", "$v_\varepsilon \in W_0$, $w_\varepsilon \in B_1$", "$v_\varepsilon \in W_1$, $w_\varepsilon \in B_0$", "$v_\varepsilon \in W_1$, $w_\varepsilon \in B_1$", respectively; $F_0(v,\cdot)$ and $F_1(v,\cdot)$ are meromorphic, null in $d_0$ having each a single pole in $v$ with residue $1/\pi$, $\mathrm{Re}F_0(v,\cdot)$ and $\mathrm{Im}F_1(v,\cdot)$ being null on $\partial U$. The final theorem is an asymptotic expression for the number of decompositions into dominoes of $P_\varepsilon$, containing $o(\log(1/\varepsilon))$ when $U$ is not a rectangle and $o(1)$ when it is. The paper contains other comments and relations with physics, particularly with quantum gravitation.

For the entire collection see [Zbl 0981.00011].

Reviewer: Ion Cuculescu (Bucureşti)

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