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Three finite classes of hypergeometric orthogonal polynomials and their application in functions approximation. (English) Zbl 1017.33005

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Classical orthogonal polynomials of Jacobi, Laguerre and Hermite are characterized as the infinite sequences of orthogonal polynomials which satisfy a second-order differential equation of the form:

$$(ax^2+bx+c)y_n''(x)+(dx+e)y_n'(x)-n[(n-1)a+d]y_n(x)=0 \qquad (n\in\mathbb{N}),$$

where a, b, c, d, e are parameters independent of n. In the paper under review, the author presents three other sequences of hypergeometric polynomials which are special solutions of the above equation and are finitely orthogonal with respect to three particular weight functions on infinite intervals. These classes are related, respectively, with the Jacobi, ultraspherical and Laguerre polynomials; in particular, the third class is directly related to the generalized Bessel polynomials and has also relation with the Laguerre polynomials. General properties of these sequences (such as orthogonality relation, Rodrigues type formula, recurrence relations, shift operators and generating functions) are indicated. Under the Dirichlet conditions, the function $f(x)$ is approximable in terms of a finite sum of each of these classes so that one can consider any arbitrary precision degree $n=N$ for the aforesaid approximations. Finally, estimating three kinds of definite integrals, using Gauss integration theory and polynomial weight functions, are introduced.

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MSC:

33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)
33C47 Other special orthogonal polynomials and functions

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