Topalov, Peter; **Matveev, Vladimir S.**
Geodesic equivalence via integrability. (English) Zbl 1017.37029
Geom. Dedicata 96, 91-115 (2003).

Given two symplectic manifolds (M, ω) and (M', ω') with Hamiltonian functions H and H', let $\Phi : M \to M'$ be a diffeomorphism mapping an isoenergy surface Q in M to an isoenergy surface Q' in M'. The diffeomorphism is said to be orbital if its restriction to Q takes the orbits of the Hamiltonian system (M, ω, H) to the orbits of (M', ω', H'). In the paper under review, the authors shows how to construct a one-paramter family of integrals from a given orbital diffeomorphism between two Hamiltonian systems.

As a significant application of this result, whenever two geodesically equivalent Riemannian metrics g and g' are given on the same manifold M, a set of $n = \dim M$ integrals for the geodesic flow is obtained. Such integrals Poisson commute as a consequence of a classical result proved by Levi-Civita in 1896. Let G be the endomorphism of the tangent bundle defined by raising one index of g' by means of g. If the eigenvalues of G are all distinct at least at one point and M is connected, then they are all distinct at almost every point, and the integrals are proved to be functionally independent. Therefore, the geodesics flows of the two metrics are completely integrable in the sense of Arnol'd-Liouville.

As a corollary of this result, the authors find out a topological condition preventing a closed connected real-analytic manifold from admitting a pair of geodesically equivalent metrics such that the eigenvalues of G are all distinct at least at one point.

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MSC:

37J35	Completely integrable finite-dimensional Hamiltonian systems, integration methods, integrability tests	Cited in **32** Documents
53C22	Geodesics in global differential geometry	
70H06	Completely integrable systems and methods of integration for problems in Hamiltonian and Lagrangian mechanics	
53A20	Projective differential geometry	

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