Gliklikh, Yu. E.; **Morozova, L. A.**

On the notion of L^1-completeness of a stochastic flow on a manifold. (English) Zbl 1018.58028

Abstr. Appl. Anal. 7, No. 12, 627-635 (2002).

Fix a finite-dimensional manifold M, $T>0$, $M^T := [0,T]\times M$, a generator A on M, and $A^T := \frac{\partial}{\partial t} + A$. For $(t,x)\in M^T$, denote by $\xi_{t,x}$ the stochastic flow on M (associated with A) such that $\xi_{t,x}(t)=x$.

The flow ξ is said to be "L^1-complete on $[0,T]$" if: it is complete on $[0,T]$; there exists a smooth proper $\nu : M\to\mathbb{R}_+$ such that $\mathbb{E}[\nu(\xi_{t,x}(T))]<\infty$ for all $(t,x)\in M^T$; for any $K>0$ there exists a compact $C_{K,T}\subset M$ such that $\mathbb{E}[\nu(\xi_{t,x}(T))]<K \Rightarrow x\in C_{K,T}$; $(t,x)\mapsto \mathbb{E}[\nu(\xi_{t,x}(T))]$ is smooth.

Then it is proved that ξ is L^1-complete on $[0,T]$ if and only if there exists a smooth proper $u : M^T\to\mathbb{R}_+$ such that: $A^T u$ is constant and for all $(t,x)\in M^T$ the variables $u(T\wedge\tau_n, \xi_{t,x}(T\wedge\tau_n))$ are uniformly integrable, where τ_n, $n\in\mathbb{N}$, denotes the time at which $s\mapsto u(s,\xi_{t,x}(s))$ exists on $[0,n]$.

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MSC:

58J65 Diffusion processes and stochastic analysis on manifolds
58J35 Heat and other parabolic equation methods for PDEs on manifolds
60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)

Keywords:

stochastic flow; stochastic completeness; stochastic L^1-completeness

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