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Some results on fixed point theorems for multivalued mappings in complete metric spaces. (English) Zbl 1020.47048

Int. J. Math. Math. Sci. 30, No. 6, 319-325 (2002).

Let (X, d) be a metric space, then a function $p : X \times X \to [0, \infty)$ is called a w-distance on X [*O. Kada*, *T. Suzuki* and *W. Takahashi*, Math. Jap. 44, 381-391 (1996; Zbl 0897.54029)] if: (1) $p(x, z) \leq p(x, y) + p(y, z)$ for all $x, y,\ z \in X$; (2) for any $x \in X$, $p(x, .) : X \to [0, \infty)$ is lower semicontinuous; (3) for any $\varepsilon > 0$, there exists $\delta > 0$ such that $p(z, x) \leq \delta$ and $p(z, y) \leq \delta$ imply $d(x, y) \leq \varepsilon$. For any $x \in X$ and $A \subset X$ we denote by $p(x, A) = \{\inf p(x, y) : y \in A)$ and by $p(A, x) = \inf\{p(y, x) : y \in A\}$ and $CB_p(X) = \{A \mid A$ is a nonempty closed susbset of X and $\sup_{x,y\in A} p(x, y)\} < \infty$.

Recently, *J.-S. Ume* [J. Math. Anal. Appl. 225, 630-640 (1998; Zbl 0917.54047)] improved some fixed points theorem in a complete metric space using the concept of w-distance. In the paper under review the authors, using this concept, prove some common fixed point theorems for two multivalued mappings S and T in a complete metric space.

The main result of this paper is the following theorem: Let X be a complete metric space with a metric d and let p be a w-distance on X. Suppose that S and T are two mappings of X into $CB_p(X)$ and $f : X \times X \to [0, \infty)$ is a mapping such that $\max\{p(u_1, u_2),\ p(v_1, v_2)\} \leq qf(x, y)$ for all nonempty subsets A, B of X, $u_1 \in SA$, $u_2 \in S^2A$, $v_1 \in TB$, $v_2 \in T^2B$, $x \in A$, $y \in B$, and some $q \in [0, 1]$ with $\sup\{\sup(f(x, y)/\min[p(x, SA),\ p(y, TB) : x \in A,\ y \in B) : A, B < X\} < 1/q$, $\inf\{p(y, u) + p(x, Sx) + p(y, Ty) : x, y \in X\} > 0$, for every $u \in X$ with $u \in Su$ or $u \notin Tu$, where SA means $\cup\{Sa : a \in A\}$. Then S and T have a common fixed point.

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MSC:

47H10 Fixed-point theorems
54H25 Fixed-point and coincidence theorems (topological aspects)
47H04 Set-valued operators

Keywords:

w-distance; common fixed point theorems; multivalued mappings; complete metric space

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