Zudilin, V. V.

One of the eight numbers $\zeta(5), \zeta(7), \cdots, \zeta(17), \zeta(19)$ is irrational. (English. Russian original)

Zbl 1022.11035
Math. Notes 70, No. 3, 426-431 (2001); translation from Mat. Zametki 70, No. 3, 472-476 (2001).

The author proves the result given in the title by refining the method used by the reviewer to prove the following weaker result: one of the nine numbers $\zeta(5), \zeta(7), \ldots, \zeta(21)$ is irrational [Acta Arith. 103, 157-167 (2002; Zbl 1015.11033)]. Using a very-well-poised series (of hypergeometric type), he constructs a sequence of linear forms $S_n = p_{0,n} + p_{1,n}\zeta(5) + \cdots + p_{8,n}\zeta(19)$ with rational coefficients: his improvement is due to his very careful study of the p-adic valuation of the integers $D_n p_{j,n}$, where D_n denotes a common denominator D_n of the $p_{j,n}$. He uses for this a "prime extracting" method introduced by Chudnovsky, Hata and others, and removes a "big" common factor Π_n of the integers $D_n p_{j,n}$. Finally, he applies the saddle point method to prove that $\Pi_n^{-1} D_n S_n$ is never 0 and tends to 0 as n tends to infinity, which proves the theorem.

He also notes that the same method proves the irrationality of at least one number in each of the sets $\{\zeta(7), \zeta(9), \ldots, \zeta(35)\}$ and $\{\zeta(9), \zeta(11), \ldots, \zeta(51)\}$.

Note finally that, in the meantime, a further refinement of this arithmetical scheme has enabled the author to prove that at least one of the four numbers $\zeta(5), \zeta(7), \zeta(9), \zeta(11)$ is irrational ["Arithmetic of linear forms involving odd zeta values", (to appear in) J. Théor. Nombres Bordx. 16, No. 1, 251–291 (2004; Zbl 1156.11327)].

Reviewer: Tanguy Rivoal (Paris)

MSC:

11J72 Irrationality; linear independence over a field
11M06 $\zeta(s)$ and $L(s, \chi)$

Keywords:

Riemann zeta-function; irrationality of values of the zeta-function; Nikishin-Rivoal arithmetical scheme

Full Text: DOI