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On the vanishing of $H_3(SL_2(A, I), \mathbb{Z}/l)$. (English) Zbl 1025.19003


The Friedlander-Milnor isomorphism conjecture says that for any connected Lie group $G$, the canonical map $BG \to BG^{top}$ induces an isomorphism on homology groups with finite (trivial) coefficients in $\mathbb{Z}/l$, where $BG$ denotes the classifying space of $G$ as a discrete group while $BG^{top}$ is the classifying space of $G$ considered as a topological group. All known approaches to this conjecture use one or another form of the so-called rigidity conjecture. The author of this paper states different versions of the rigidity conjecture for arbitrary reductive algebraic groups $G$. The most interesting case is that of $SL_n$ and especially of $SL_2$.

Let $G$ be a split reductive algebraic group over a field $F$, $A$ be a local henselian $F$-algebra with maximal ideal $I$, and $l \geq 1$ be an integer prime to the characteristic of $F$. The author constructs a Hochschild-Serre spectral sequence for non-normal subgroups, which degenerates, providing a relatively small complex computing homology of the congruence subgroup. By using this complex and its modifications, the author is able to compute the homology of $SL_2(A, I)$. In particular, it is proved that $H_3(SL_2(A, I), \mathbb{Z}/l)$ is trivial in the case $l = 2$.

For the entire collection see [Zbl 1013.00021].

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MSC:

19D55 $K$-theory and homology; cyclic homology and cohomology
14F17 Vanishing theorems in algebraic geometry
20G15 Linear algebraic groups over arbitrary fields

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