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On Hurwitzian and Tasoev's continued fractions. (English) Zbl 1026.11012
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A Hurwitzian continued fraction (CF) is defined to be

$$\left[c_0; c_1, \ldots, c_n, \overline{Q_1(k), \ldots, Q_p(k)}\right]_{k=1}^{\infty},$$

where c_i are integers ($c_i \geq 1$ if $i \geq 1$) and $Q_j(k)$ are polynomials in k taking positive integer values for $k = 1, 2, \ldots$. Tasoev's CF has the same form as the Hurwitzian CF where $Q_j(k)$ may contain exponential terms in k. In this paper, Tasoev's CF $\left[0; \overline{ua^k}\right]_{k=1}^{\infty}$ and $\left[0; \overline{ua^k, va^k}\right]_{k=1}^{\infty}$ are represented in a closed form which is a fraction of two infinite series. E.g.

$$\left[0; \overline{ua^k}\right]_{k=1}^{\infty} = \frac{\sum_{s=0}^{\infty}(u^{2s+1}a^{(s+1)^2}\prod_{i=1}^{s}(a^{2i}-1))^{-1}}{\sum_{s=0}^{\infty}(u^{2s}a^{s^2}\prod_{i=1}^{s}(a^{2i}-1))^{-1}}.$$

Also closed form representations of

$$\left[0; ua-1, \overline{1, ua^{k+1}-2}\right]_{k=1}^{\infty} \quad \text{and} \quad \left[0; ua-1, 1, va-2, \overline{1, ua^{k+1}-2, 1, va^{k+1}-2}\right]_{k=1}^{\infty}$$

are given. The same method is applied to the Hurwitzian CF

$$\left[0; \overline{u(c+(2k-2)d), v(c+(2k-1)d)}\right]_{k=1}^{\infty}$$

and

$$\left[0; uc-1, 1, v(c+d)-2, \overline{1, u(c+2kd)-2, 1, v(c+(2k+1)d)-2}\right]_{k=1}^{\infty}$$

to get e.g.

$$\left[0; \overline{u(c+(2k-2)d), v(c+(2k-1)d)}\right]_{k=1}^{\infty} = \frac{\sum_{s=0}^{\infty}(s!u^{s+1}(vd)^s\prod_{i=0}^{s}(c+id))^{-1}}{\sum_{s=0}^{\infty}(s!(uvd)^s\prod_{i=0}^{s-1}(c+id))^{-1}}.$$

Reviewer: M.Ohtsuki (Kodaira)

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11J70 Continued fractions and generalizations

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Hurwitzian continued fraction; Tasoev continued fraction; negative continued fraction

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