Carlehed, Magnus; Wiegerinck, Jan

Le cône des fonctions plurisousharmoniques négatives et une conjecture de Coman. (A cone of plurisubharmonic negative functions and a conjecture of Coman). (French) Zbl 1026.32066

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The negative plurisubharmonic functions on a domain Ω in $\mathbb{C}^n$ form a convex cone in the space of locally integrable functions on Ω. The authors give a few new examples of functions that are extremal points in this cone. These examples are pluricomplex Green functions with two poles. If w_j, $j = 1, \dots, k$ are finitely many points in Ω, ν_j are positive numbers, and $A = \{(w_j, \nu_j); j = 1, \dots, k\}$, then the pluricomplex Green function $g(\cdot, A)$ with weights ν_j at w_j is defined as the supremum of all negative plurisubharmonic functions u such that $u - \nu_j \log |\cdot - w_j|$ is bounded near w_j for each j. The points w_j are called the poles of the function $g(\cdot, A)$.

The authors prove that the Green function with two poles of equal weight 1 in the unit ball are extremal as well as the Green function with the poles $(a, 0)$ and $(b, 0)$ of equal weight 1 in the bidisc. The Lempert function $\delta(\cdot, A)$ with poles of weight ν_j at w_j is defined so that $\delta(z, A)$ is the infimum of sums of the form $\sum_{j=1}^{k} \nu_j \log |\zeta_j|$ taken over all analytic discs $f : D \to \Omega$ such that $f(0) = z$ and $f(\zeta_j) = w_j$. *D. Coman* studied these functions in his paper [Pac. J. Math. 194, 257-283 (2000; Zbl 1015.32029)] and conjectured that $g(\cdot, A) = \delta(\cdot, A)$ for every A on any bounded convex domain. The authors show that for the bidisc with the points $(a, 0)$ and $(b, 0)$, $0 < |a|, |b| < 1$, with the weights 1 and 2, these functions are not equal. They even prove the existence of strictly pseudoconvex domains in the bidisc for which the functions are different.

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MSC:

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32U05 Plurisubharmonic functions and generalizations

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