Linial, Nathan; Samorodnitsky, Alex
Linear codes and character sums. (English) Zbl 1026.94024
Combinatorica 22, No. 4, 497-522 (2002).

G. Kalai and N. Linial [IEEE Trans. Inf. Theory 41, 1467-1472 (1995; Zbl 0831.94019)] conjectured that the size of the code with the distribution of distances near the minimal distance is exponentially small. The authors estimate the fraction of non-zero vectors of minimal weights in an $r \cdot n$-dimensional subspace of $\mathbb{Z}_2^n$. Using the connection between the value distributions of character sums over $\mathbb{Z}_2^n$ and extremal problems on linear codes the authors prove that for $r \gg \frac{\log n}{\sqrt{n}}$ the above fraction is exponentially small and does not exceed $2^{-\Omega(\frac{r^2}{\log(1/r) + 1} n)}$.

Let $E$ be the affine subspace in $\mathbb{Z}_2^n$ of dimension $a \cdot n (a > \frac{1}{2}, n$ is even and large) and $L_k$ be the set of vectors in $\mathbb{Z}_2^n$ whose Hamming weight is $k$. The authors also answer a question of M. Ben-Or (preprint of Hebrew University, 1996) about the largest possible number of a given weight. They prove that for every $0 \leq k \leq n$ the inequality

$$|E \cap L_k| \leq C_a \frac{|E|}{\sqrt{n}}$$

is satisfied.

Reviewer: Piroska Lakatos (Debrecen)

MSC:
94B65 Bounds on codes
05D05 Extremal set theory
94B99 Theory of error-correcting codes and error-detecting codes

Keywords:
Hamming weight; fraction of code words with a fixed weight; character sums

Full Text: DOI