Piccione, Paolo; Tausk, Daniel V.

An index theorem for non-periodic solutions of Hamiltonian systems. (English)


There exist in the literature some index theories for Hamiltonian systems that mostly concern the case of periodic solutions. The purpose of the present paper is a theory for an index form \( I_\Gamma \) associated to a general Hamiltonian set-up consisting of a symplectic manifold \( (\mathcal{M}, \omega) \), a Lagrangian submanifold \( \mathcal{P} \) of \( \mathcal{M} \), a distribution \( \mathcal{L} \) of Lagrangian subspaces in \( \mathcal{M} \), a time-dependent Hamiltonian function \( H \) on \( \mathcal{M} \) and a given solution \( \Gamma : [a, b] \to \mathcal{M} \) of Hamilton equations with \( \Gamma(a) \in \mathcal{P} \). For example, if \( \mathcal{M} \) is the cotangent bundle \( T^*\mathcal{M} \) of a manifold \( \mathcal{M} \), the distribution \( \mathcal{L} \) is the vertical bundle of \( TT^*\mathcal{M} \) and \( H \) is a hyper-regular Hamiltonian then \( I_\Gamma \) is exactly the second variation of the Lagrangian action functional.

The main result of this paper is an index theorem, namely Theorem 2.7.10, that relates the index of a suitable restriction of \( I_\Gamma \) to the Maslov index of a symplectic differential system. Therefore, a large presentation of symplectic differential systems and applications to Hamiltonian systems and semi-Riemannian geometry is included. Finally, a geometrical application of the index theory to the geodesic Hamiltonian for semi-Riemannian geometry is presented.

Reviewer: Mircea Crăşmăreanu (Iasi)

MSC:

- 53D12 Lagrangian submanifolds; Maslov index
- 37J05 Relations of dynamical systems with symplectic geometry and topology (MSC2010)
- 58E10 Variational problems in applications to the theory of geodesics (problems in one independent variable)
- 53D25 Geodesic flows in symplectic geometry and contact geometry

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