Rivoal, T.; **Zudilin, W.**

Diophantine properties of numbers related to Catalan's constant. (English) Zbl 1028.11046
Math. Ann. 326, No. 4, 705-721 (2003).

The main subject of this paper is to study the arithmetic nature of numbers related to Dirichlet's β-function:

$$\beta(s) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^s}.$$

Lindemann's theorem implies that $\beta(2n+1)$ is transcendental for $n \in \mathbb{N}$. The authors consider the case of even integers and prove:

Theorem. Let a be an even positive integer and let $\delta(a)$ be the dimension of the $\mathbb{Q}$-vector space spanned by the numbers 1, $\beta(2)$, $\beta(4)$, ..., $\beta(a)$, then

$$\delta(a) \geq \frac{1+o(1)}{2+\log(2)} \log(a) \qquad \text{as } a \to \infty.$$

Their method is effective and they show that at least one of the numbers $\beta(2)$, $\beta(4)$, ..., $\beta(14)$ is irrational. The main tool in the proof is the study of well-chosen hypergeometric functions. This study is quite delicate and detailed. The proof is about 15 pages long.

Reviewer: Maurice Mignotte (Strasbourg)

MSC:

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Full Text: DOI Link