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**On some sharp conditions for lower semicontinuity in  $L^1$ .** (English) Zbl 1028.49012  
Differ. Integral Equ. 16, No. 1, 51-76 (2003).

Summary: Let  $\Omega$  be an open set of  $\mathbb{R}^n$  and let  $f : \Omega \times \mathbb{R} \times \mathbb{R}^n$  be a nonnegative continuous function, convex with respect to  $\xi \in \mathbb{R}^n$ . Following the well-known theory originated by *J. Serrin* [Trans. Am. Math. Soc. 101, 139-167 (1961; [Zbl 0102.04601](#))], we deal with the lower semicontinuity of the integral

$$F(u, \Omega) = \int_{\Omega} (x, u(x), Du(x)) dx$$

with respect to the  $L^1_{\text{loc}}(\Omega)$  strong convergence. Only recently it has been discovered that dependence of  $f(x, s, \xi)$  on the  $x$  variable plays a crucial role in the lower semicontinuity. In this paper we propose a mild assumption on  $x$  that allows us to consider discontinuous integrands, too. More precisely, we assume that  $f(x, s, \xi)$  is a nonnegative Carathéodory function, convex with respect to  $\xi$ , continuous in  $(s, \xi)$  and such that  $f(\cdot, s, \xi) \in W^{1,1}_{\text{loc}}(\Omega)$  for every  $s \in \mathbb{R}$  and  $\xi \in \mathbb{R}^n$ , with the  $L^1$  norm of  $f_x(\cdot, s, \xi)$  locally bounded. We also discuss some other conditions on  $x$ ; in particular we prove that Hölder continuity of  $f$  with respect to  $x$  is not sufficient for lower semicontinuity, even in the one dimensional case, thus giving an answer to a problem posed by the authors in [J. Convex Anal. 9, No. 2, 475-502 (2002; [Zbl 1019.49021](#))]. Finally, we investigate the lower semicontinuity of the integral  $F(u, \Omega)$ , with respect to the strong norm topology of  $L^1_{\text{loc}}(\Omega)$ , in the vector-valued case, i.e., when  $f : \Omega \times \mathbb{R}^m \times \mathbb{R}^{m \times n} \rightarrow \mathbb{R}$  for some  $n \geq 1$  and  $m > 1$ .

**MSC:**

[49J45](#) Methods involving semicontinuity and convergence; relaxation

[49J10](#) Existence theories for free problems in two or more independent variables

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Cited in **7** Documents

**Keywords:**

[lower semicontinuity](#); [Hölder continuity](#)