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Equivalent formulations of Ekeland’s variational principle. (English) Zbl 1029.49019

Summary: We prove that for some $0 < \alpha$ and $0 < \varepsilon \leq +\infty$ a proper lower semicontinuous and bounded below function $f$ on a metric space $(X,d)$ satisfies that for each $x \in X$ with $\inf_{X} f < f(x) < \inf_{X} f + \varepsilon$ there exists $y \in X$ such that $0 < \alpha d(x,y) \leq f(x) - f(y)$ iff for each such $x$ this inequality holds for some minimizer $z$ of $f$. Similar conditions are shown to be sufficient for $f$ to possess minimizers, weak sharp minima and error bounds. A fixed-point theorem is also established. Moreover, these results all turn out to be equivalent to the Ekeland variational principle, the Caristi–Kirk fixed-point theorem and the Takahashi theorem.

MSC:
49J53 Set-valued and variational analysis
49J27 Existence theories for problems in abstract spaces
54H25 Fixed-point and coincidence theorems (topological aspects)

Keywords:
Ekeland’s variational principle; $\varepsilon$-condition of Takahashi; $\varepsilon$-condition of Hamel; weak sharp minima; error bounds; fixed point theorem

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References:

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