**Heath-Brown, D. R.**; **Moroz, B. Z.**
**Primes represented by binary cubic forms.** (English) Zbl 1030.11046
Proc. Lond. Math. Soc. (3) 84, No. 2, 257-288 (2002).

Let $f(x,y)$ be a binary cubic form with integral rational coefficients, and suppose that the polynomial $f(x,y)$ is irreducible in $\mathbb{Q}[x,y]$ and no prime divides all the coefficients of $f$. We prove that the set $f(\mathbb{Z}^2)$ contains infinitely many primes unless $f(a,b)$ is even for each $(a,b)$ in $\mathbb{Z}^2$, in which case the set $\frac{1}{2}f(\mathbb{Z}^2)$ contains infinitely many primes. This theorem is a generalization of the recent theorem of *D. R. Heath-Brown* [Acta Math. 186, 1–84 (2001; Zbl 1007.11055)] on the infinitude of the primes represented by the cubic form $x^3+2y^3$, and its proof follows the pattern set up in that paper. The main innovations are related to the arithmetic of a general cubic field; our considerations require, in particular, some techniques from E. Hecke's multidimensional arithmetic which we briefly review. As in the cited work of Heath-Brown, we have actually obtained an asymptotic formula for the relevant number of primes.

Reviewer: B.Z.Moroz (Bonn)

**MSC:**

| | |
|---|---|
| 11N32 | Primes represented by polynomials; other multiplicative structures of polynomial values |
| 11N36 | Applications of sieve methods |
| 11R44 | Distribution of prime ideals |



**Keywords:**
primes; binary cubic forms; grossencharacters

**Full Text:** DOI