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Estimates for moduli of coefficients of positive trigonometric polynomials. (English)

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Sci. Math. Jpn. 56, No. 1, 115-122 (2002).

The authors reprove the following theorem of *G. Szegö* [Math. Ann. 96, 601-632 (1927; JFM 53.0465.04)] and *E. Egerváry* and *O. Szász* [Math. Z. 27, 641-652 (1928; JFM 54.0314.01)]: If $T(\theta) = 1+\sum_{k=1}^{n}(a_k \cos k\theta + b_k \sin k\theta)$ is a nonnegative trigonometric polynomial, then $\sqrt{a_k^2 + b_k^2} \leq 2\cos\frac{\pi}{[n/k]+2}$, where $1 \leq k \leq n$. Moreover, the equality is attained if and only if $T(\theta)$ is of the form

$$\tau(\theta)\left\{1 + \frac{2}{2+p}\sum_{\nu=1}^{p}\left((p-\nu+1)\cos\nu\alpha + \frac{\sin(\nu+1)\alpha}{\sin\alpha}\right)\cos(\nu k(\theta-\psi))\right\},$$

where τ is an arbitrary nonnegative trigonometric polynomial of order q, $\alpha = \pi/(p+2)$, $p = [n/k]$, $n = pk+q$, $0 \leq q < k$, and ψ is an arbitrary constant.

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MSC:

42A32	Trigonometric series of special types (positive coefficients, monotonic coefficients, etc.)	Cited in **1** Review
47A12	Numerical range, numerical radius	
15A60	Norms of matrices, numerical range, applications of functional analysis to matrix theory	

Keywords:
positive trigonometric polynomial; numerical radius; shift matrix