**Christodoulou, Demetrios**; **Lindblad, Hans**
**On the motion of the free surface of a liquid.** (English) Zbl 1031.35116
Commun. Pure Appl. Math. 53, No. 12, 1536-1602 (2000).

This is a very solid and deep paper dealing with the free boundary problem of an incompressible perfect fluid. The velocity $v$ of the fluid obeys the Euler equations inside the moving domain and equals the normal velocity of the boundary on the free boundary. The directional derivative of the pressure $p$ along the exterior unit normal to the boundary is assumed to be negative, namely $\nabla_{\mathcal{N}} p \leq -\varepsilon < 0$. This is a natural physical condition since the pressure has to be positive in the interior of the fluid. The major purpose of this paper is to establish global a priori bounds in Sobolev spaces for this free boundary problem supplemented with an initial datum for $v$. It is worth pointing out that the results of this paper hold for the case when the vorticity of the fluid is not zero.

To prove their results, the authors introduce a sequence of energies $\{E_r\}$, each of which consists of an interior part and a boundary part. The advantage of such energies is to avoid the use of fractional Sobolev spaces. The authors then show that the time derivative of $E_r$ is bounded by a constant $C$ times $\sum_{s=0}^{r} E_s$, where $C$ depends on $1/\varepsilon$, and the geometry and regularity of the free boundary. A crucial fact in obtaining this bound is that the time derivative of the interior part, after integration by parts, cancels the leading order term in the time derivative of the boundary part. The global a priori bounds for $E_r$ are then proved iteratively. In the course of the proof, significant efforts are devoted to controlling the geometry and regularity of the boundary. For this purpose, the authors work in both Eulerian coordinates and the Lagrangian coordinates.

Reviewer: Jiahong Wu (Stillwater)

**MSC:**

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|---|---|
| 35Q35 | PDEs in connection with fluid mechanics |
| 76U05 | General theory of rotating fluids |
| 76D05 | Navier-Stokes equations for incompressible viscous fluids |
| 76D03 | Existence, uniqueness, and regularity theory for incompressible viscous fluids |



**Keywords:**

Navier-Stokes equation; QG equations

**Full Text:** DOI

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