Bahturin, Yu. A.; Parmenter, M. M.
Generalized commutativity in group algebras. (English) Zbl 1032.16023

This is an interesting paper. It considers various $F$-algebras and asks whether they can be graded by an Abelian group $\Gamma$ in such a way that the algebra becomes $\beta$-commutative for some bicharacter $\beta : \Gamma \times \Gamma \to F^*$. For example, they use representations of nilpotent of class 2 groups to show that if the field $F$ contains a primitive $n$-th root of unity, then the matrix algebra $M_n(F)$ can be so graded. On the other hand, this is not the case when $\text{char} \ F = p > 0$ and $p$ divides $n$. It follows from the above that if $F$ is an algebraically closed field and if $|G| \neq 0$ in $F$, then the group algebra $F[G]$ admits a $\beta$-commutative grading. On the other hand, it is shown here that if $\text{char} \ F = p > 0$ and if $G$ is a finite $p$-group, then $F[G]$ can be $\beta$-commutative if and only if $G$ is Abelian. Finally, the authors consider group algebras of infinite groups. Here the starting point is the fact that if $F[G]$ is $\beta$-commutative, then it must satisfy a polynomial identity. Hence $G$ must have a reasonable structure.

Reviewer: Donald S.Passman (Madison)

MSC:
16S34 Group rings
16W50 Graded rings and modules (associative rings and algebras)
16R50 Other kinds of identities (generalized polynomial, rational, involution)
16U80 Generalizations of commutativity (associative rings and algebras)
20C07 Group rings of infinite groups and their modules (group-theoretic aspects)

Keywords:
group algebras; graded rings; bicharacters; matrix algebras; polynomial identities

Full Text: DOI