Hehl, Friedrich W.; Obukhov, Yuri N.
Foundations of classical electrodynamics. Charge, flux, and metric. (English) Zbl 1032.78001

The object of this book, according to the authors’ preface, is to provide a textbook of classical electromagnetic theory, the approach resting “on the metric-free integral of the formulation of the conservation laws of electrodynamics” and “the automatic point of view”. This, they believe, gives a better understanding of the structure of electrodynamics as a classical field theory, thereby allowing a smoother transition to the ideas of general relativity. This is done by means of a differential form of approach.

The book opens with a short introduction in which the axioms of electromagnetism are discussed. The first proper section, Part A, is concerned with introducing the ideas of the exterior calculus. There is a short discussion on the advisability of using exterior differential forms, a treatment of their algebra, including tensors and some of their generalizations, and other exterior alternatives. This leads on to manifold theory and De Rham’s theorems. In Part B, the authors deal with the axioms of classical electrodynamics which describe the phenomenological facts. The conservation laws are formulated and invariants of electromagnetic fields are obtained and it is shown that the law of absence of magnetic charge follows from one of the axioms. Figures are given of flux lines in a type II superconductor and in the core of a computer-simulated earth. There is a treatment of the boundary conditions on the fields at the interface between two media. Other topics considered are Lenz’s rule, local noninertial frames and the quantum Hall effect. Finally, an axiom associated with the energy-momentum tensor is formulated, a Lagrangian analysis is developed and the idea of action of an electromagnetic field is mentioned.

Part C is entitled “More Mathematics” and the authors commence by discussing the ideas of linear connection and covariant differentiation. Following on this, the authors comment on each topic as torsion curvature and metric. Hodge stars are introduced and there is a short comment on non-metricity. In distinction to the other parts, this part closes with some problems (there appears to be something wrong in the problems C4/3-4 as \( L \neq SR \)). Part D has the title “The Maxwell-Lorentz spacetime relations”. This begins by showing that the relation between excitation and field strength, if assumed linear, involves a tensor with 36 independent components. The nature of these is analysed and the nature of the propagation of plane electromagnetic waves is discussed. A Lagrangian is introduced and a relation is obtained between the light cone and a quartic wave surface. Part E is entitled “Electrodynamics in vacuum and in matter”. In this, the standard electromagnetic theory for a vacuum is considered, including a discussion of the mechanical effects associated with the electromagnetic field. An axiom is introduced to deal with the fields within matter and some effects of dispersion and nonlinearity are considered. Modifications of Maxwell’s equations needed for moving matter are developed including the effect of non-inertial frames of reference. The text closes with a part “outlook”. In this the authors discuss the relation between gravity and electrodynamics, make some comments on topological connections and mention modern developments.

Throughout this book, the rationalized MKS system of units is used, making analysis more intelligible and there are many diagrams which are of great help in understanding the text. Each part of the book is followed by a copious list of references, over 300 in all, about 10% not being in English. Also, in appropriate places there are indications how computer algebra (REDUCE/EXCALC) can be used. There is an index which is satisfactory on the whole, although a number of terms (e.g. coframes, axioms) are introduced earlier in the book than as stated in the index. One difficulty with this work is that in a number of places, the analysis leading up to a formula e.g. CZ.120 is inadequate or non existent.

Generally, this book will not be of much use to those working in the fields of say plasmas, waveguide or circuits being too abstract, but will be useful to (very) theoretical physicists. Perhaps a better title would be something like “Exterior Calculus in Electromagnetic Theory”. It does however give an interesting approach to the theory. The printing and appearance of the book are excellent, but it is unfortunately somewhat on the expensive side. It can be warmly recommended.

Reviewer: Ll.G.Chambers (Bangor)
MSC:
78-02 Research exposition (monographs, survey articles) pertaining to optics and electromagnetic theory
83C50 Electromagnetic fields in general relativity and gravitational theory
53Z05 Applications of differential geometry to physics
81-02 Research exposition (monographs, survey articles) pertaining to quantum theory
85A40 Astrophysical cosmology

Keywords:
classical electrodynamics; charge; flux metric

Software:
EXCALC