Bost, Jean-Benoît

Algebraic leaves of algebraic foliations over number fields. (English) Zbl 1034.14010

Publ. Math., Inst. Hautes Étud. Sci. 93, 161-221 (2001).

This paper proves an algebraicity criterion for leaves of algebraic foliations over number fields.

Let K be a number field embedded in $\mathbb{C}$, let X be a smooth algebraic variety over K (i.e., an integral separated scheme of finite type over K), and let F be an algebraic subbundle of the tangent bundle T_X. We assume that F is involutive; i.e., closed under the Lie bracket. Then F defines a holomorphic foliation of the complex manifold $X(\mathbb{C})$.

This paper proves that the leaf $\mathcal{F}$ through a rational point $P \in X(K)$ is algebraic if the following local conditions are satisfied:

(i) for almost all prime ideals $\mathfrak{p}$ of the ring of integers $\mathfrak{O}_K$ of K, the p-curvature of the reduction modulo $\mathfrak{p}$ of the subbundle $F \subseteq T_X$ vanishes at P (here p is the prime of $\mathbb{Z}$ lying below $\mathfrak{p}$); and

(ii) the manifold $\mathcal{F}$ satisfies the Liouville property: Every plurisubharmonic function on $\mathcal{F}$ bounded from above is constant. [For example, $\mathcal{F}$ satisfies the Liouville property if it is a holomorphic image of a complex algebraic variety minus a closed analytic subset.]

These conditions are (in the words of the author) "almost necessary", in the sense that (ii) is necessary, and for almost all $\mathfrak{p}$, the p-curvature vanishes at the point where P meets the closed fiber at $\mathfrak{p}$.

As an application, let G be an algebraic group over K. Then a K-Lie subalgebra $\mathbf{h}$ of $\operatorname{Lie} G$ is algebraic if and only if for almost all primes $\mathfrak{p}$ of $\mathcal{O}_K$, the reduction modulo $\mathfrak{p}$ of $\mathbf{h}$ is a restricted Lie subalgebra of the reduction modulo $\mathfrak{p}$ of $\operatorname{Lie} G$ (i.e., it is stable under p-th powers). This solves an (unpublished) conjecture of *T. Ekedahl* and *N. Shepherd-Barron*.

The proof relies on "transcendence techniques", except that the traditional construction of auxiliary polynomials is replaced by the use of "slopes" for hermitian vector bundles over arithmetic curves, via the language of Arakelov geometry. This method was introduced by the author's work building on the work of Masser and Wüstholz [see *J.-B. Bost*, Astérisque 237, 115–161 (1996; Zbl 0936.11042)].

Reviewer: Paul Vojta (Berkeley)

MSC:

14G40	Arithmetic varieties and schemes; Arakelov theory; heights
11G35	Varieties over global fields
14G25	Global ground fields in algebraic geometry
11J81	Transcendence (general theory)
14L30	Group actions on varieties or schemes (quotients)

Keywords:

algebraicity; foliation; Arakelov geometry; p-curvature; slope

Full Text: DOI Numdam EuDML