Zhang, Shouwu

Heights of Heegner points on Shimura curves. (English) Zbl 1036.11029

Ann. Math. (2) 153, No. 1, 27-147 (2001).

Let F be a totally real number field, and let N be a nonzero ideal of the ring $\mathcal{O}_F$ of integers in F. Let f be a new form on $\mathrm{GL}_2(\mathbb{A}_F)$ of weight two and level $K_0(N)$ with trivial central character, where $K_0(N) = \{\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{GL}(\mathcal{O}_F \times \prod_p \mathbb{Z}_p) \mid c \in N \times \prod_p \mathbb{Z}_p\}$. Let $\mathcal{O}_f$ be the subalgebra of $\mathbb{C}$ over $\mathbb{Z}$ generated by the eigenvalues $a(m, f)$ of f under the Hecke operators $T(m)$ with $(m, N) = 1$. Each embedding $\sigma : \mathcal{O}_f \to \mathbb{C}$ determines a new form f^σ such that $a(f^\sigma, m) = a(f, m)^\sigma$. Assume that either $[F : \mathbb{Q}]$ is odd of $\mathrm{ord}_v(N) = 1$ for at least one finite place v of F. Then there exists an abelian variety A over F of dimension $[\mathcal{O}_f : \mathbb{Z}]$ such that its L-function $L(s, A)$ coincides with $\prod_{\sigma:\mathcal{O}_f\to\mathbb{C}} L(s, f^\sigma)$ modulo the factors at the places dividing N. In this paper, under the assumption that the L-function $L(s, f)$ has order at most one at $s = 1$, the author proves that the rank of the Mordell-Weil group $A(F)$ is equal to $[\mathcal{O}_f : \mathbb{Z}]\mathrm{ord}_{s=1}L(s, f)$ and that the Shafarevich-Tate group of A is finite. The proof is carried out by studying Heegner points over an imaginary quadratic extension of F.

Reviewer: Min Ho Lee (Cedar Falls)

MSC:

11G18	Arithmetic aspects of modular and Shimura varieties
14G35	Modular and Shimura varieties
11F11	Holomorphic modular forms of integral weight
11G40	L-functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture
11G50	Heights

Keywords:

Shimura curves; Heegner points; L-functions; modular forms; Hecke operators; Mordell group; Shafarevich-Tate group

Full Text: DOI EuDML arXiv