

[Ogawa, Takayoshi](#)

Sharp Sobolev inequality of logarithmic type and the limiting regularity condition to the harmonic heat flow. (English) [Zbl 1036.35082](#)
[SIAM J. Math. Anal.](#) 34, No. 6, 1318-1330 (2003).

The logarithmic Sobolev inequality in scale of Lizorkin-Triebel spaces (spaces $F_{p,r}^s$) is established. The inequality is applied to the regularity problem of the smooth harmonic heat flow into a sphere,

$$u_t - \Delta u = u|\nabla u|^2, \quad u(0, x) = u_0(x),$$

where $u(t, x) : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow S^m$. The main result is the possibility of extending the solution after $t = T$ under the condition

$$\int_0^T \|\nabla u(t)\|_{\text{BMO}(\mathbb{R}^n)}^2 dt < \infty.$$

Reviewer: [Evgeniy A. Kalita \(Donetsk\)](#)

MSC:

- [35K55](#) Nonlinear parabolic equations
- [58E20](#) Harmonic maps, etc.
- [58J35](#) Heat and other parabolic equation methods for PDEs on manifolds
- [46E30](#) Spaces of measurable functions (L^p -spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)
- [46E35](#) Sobolev spaces and other spaces of “smooth” functions, embedding theorems, trace theorems
- [26D10](#) Inequalities involving derivatives and differential and integral operators

Cited in **31** Documents

Keywords:

[logarithmic inequality](#); [Lizorkin-Triebel spaces](#); [interpolation inequality](#); [regularity criterion](#); [bounded mean oscillation](#)

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