

**Benedetto, John J.; Treiber, Oliver M.**

**Wavelet frames: Multiresolution analysis and extension principles.** (English) [Zbl 1036.42032](#)  
Debnath, Lokenath, Wavelet transforms and time-frequency signal analysis. Boston, MA: Birkhäuser (ISBN 0-8176-4104-1/hbk). Applied and Numerical Harmonic Analysis, 3-36 (2001).

Frame multiresolution analysis was introduced by *J. J. Benedetto* and *S. Li* [Appl. Comput. Harmon. Anal. 5, 389–427 (1998; [Zbl 0915.42029](#))]. For  $f \in L^2(\mathbb{R})$ , let  $(T_a f)(x) = f(x - a)$  and  $(D_a f)(x) = a^{-1/2} f(\frac{x}{a})$ ,  $a \neq 0$ . An FMRA for  $L^2(\mathbb{R})$  consists of a sequence of closed subspaces  $\{V_j\}_{j \in \mathbb{Z}} \subseteq L^2(\mathbb{R})$  and a function  $\phi \in V_0$  such that (i)  $\cdots V_{-1} \subseteq V_0 \subseteq V_1 \cdots$ , (ii)  $\overline{\cup_j V_j} = L^2(\mathbb{R})$  and  $\overline{\cap_j V_j} = 0$ , (iii)  $f \in V_j \Leftrightarrow [t \rightarrow f(2t)] \in V_{j+1}$ , (iv)  $f \in V_0 \Rightarrow T_k f \in V_0$ ,  $\forall k \in \mathbb{Z}$ , and (v)  $\{T_k \phi\}_{k \in \mathbb{Z}}$  is a frame for  $V_0$ .

The purpose of an FMRA is to construct wavelet frames for  $L^2(\mathbb{R})$ , i.e., frames of the type  $\{D_{2^j} T_k \psi\}_{j,k \in \mathbb{Z}}$ ; the obvious idea is to proceed as in the construction of an orthonormal basis via an MRA. Let  $W_j$  denote the orthogonal complement of  $V_j$  in  $V_{j+1}$ . The main question is to find a wavelet  $\psi$  such that  $\{T_k \psi\}_{k \in \mathbb{Z}}$  is a frame for  $W_0$ ; this implies that  $\{D_{2^j} T_k \psi\}_{j,k \in \mathbb{Z}}$  is a frame for  $L^2(\mathbb{R})$ . Here the authors prove that the existence of such a function  $\psi$  depends solely on the “size” of the set  $\Gamma := \{x \in [0, 1] : \Phi(2x) = 0, \Phi(x) > 0, \Phi(x + \frac{1}{2}) > 0\}$ , where  $\Phi(x) = \sum_{k \in \mathbb{Z}} |\hat{\phi}(x + k)|^2$ . In fact, there exists a function  $\psi \in L^2(\mathbb{R})$  such that  $\{T_k \psi\}_{k \in \mathbb{Z}}$  is a frame for  $W_0$  if and only if  $\Gamma$  has vanishing Lebesgue measure. In case  $\Gamma$  has vanishing Lebesgue measure, the authors also show how to define a suitable function  $\psi$ .

It is explained how overcompleteness of frames implies robustness against noise that appear for example via transmission. Furthermore, a new direct proof of Ron and Shen’s Unitary Extension Principle is given.

For the entire collection see [\[Zbl 0996.00017\]](#).

Reviewer: Ole Christensen (Lyngby)

**MSC:**

- [42C40](#) Nontrigonometric harmonic analysis involving wavelets and other special systems
- [65T60](#) Numerical methods for wavelets

Cited in **1** Review  
Cited in **39** Documents

**Keywords:**

wavelet frames; frame multiresolution analysis; unitary extension principle