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Variational symmetries and Lie reduction for Frobenius systems of even rank. (English)

Zbl 1036.58004
Mladenov, Ivaïlo M. (ed.) et al., Proceedings of the 4th international conference on geometry, integrability and quantization, Sts. Constantine and Elena, Bulgaria, June 6–15, 2002. Sofia: Coral Press Scientific Publishing (ISBN 954-90618-4-1/pbk). 178-192 (2003).

Let $\mathcal{I}$ be a Frobenius system (i.e., a completely integrable Pfaffian system), $\pi \in \mathcal{I} \wedge \mathcal{I}$ a closed two-form of maximal possible rank. The author deals with relations between infinitesimal symmetries X of π (defined by the property $\mathcal{L}_X \pi = 0$) and certain reductions of the system $\mathcal{I}$. In more detail, the system $\omega_1 = \cdots = \omega_{r+s} = 0$ is called reducible to the system $\omega_1 = \cdots = \omega_r = 0$ if $d\omega_i = 0 \pmod{\omega_1, \dots, \omega_{i-1}}$ for all $r+1 \le i \le r+s$.

Theorems. Let $\mathfrak{g}$ be a solvable Lie algebra of infinitesimal symmetries of $\mathcal{I}$. $\mathcal{I}$ is reducible to the Frobenius system $\mathcal{I}(\mathfrak{g}) = \{\omega \in \mathcal{I} : \omega(X) = 0 \text{ for all } X \in \mathfrak{g}\}$ and if $\mathcal{I}(\mathfrak{g}) = 0$, then $\mathcal{I}$ is solvable by quadratures. If $\mathcal{I}$ is a rank $2k$ Frobenius system and X an infinitesimal symmetry of π, then X is infinitesimal symmetry of $\mathcal{I}$. A one-to-one correspondence exists between certain equivalence classes of infinitesimal symmetries of π and equivalence classes of conservation laws of $\mathcal{I}$.

For the entire collection see [Zbl 1008.00022].

Reviewer: Jan Chrastina (Brno)

MSC:

58A15 Exterior differential systems (Cartan theory)
58A17 Pfaffian systems
34C14 Symmetries, invariants of ordinary differential equations
34A26 Geometric methods in ordinary differential equations

Keywords:

Frobenius system; infinitesimal symmetry; solvable Lie group