Petukhov, Alexander

Symmetric framelets. (English) Zbl 1037.42038

Constructive Approximation 19, No. 2, 309-328 (2003).

Given a separable Hilbert space H, a frame is a sequence $\{e_j : j \in \mathbb{Z}\}$ of elements of H, such that there exist constants $0 < A \leq B < \infty$ such that for all $f \in H$,

$$A\|f\|^2 \leq \sum_{j\in\mathbb{Z}} |\langle f, e_j\rangle|^2 \leq B\|f\|^2.$$

A frame is tight if $A = B$. It is known that the problem of constructing tight frames of wavelets (i.e., frames of the form $\{\psi^l_{j,k}(x) = 2^{j/2}\psi^l(2^j x - k) : j, k \in \mathbb{Z}, l = 1, \ldots, n\}$) which are generated by a multiresolution analysis, can be reduced to finding functions m_k, $k = 0, \ldots, n$, which satisfy the equation

$$M(\omega)M^*(\omega) = Id,$$

where $M(\omega)(1, k) = m_k(\omega)$ and $M(\omega)(2, k) = m_k(\omega + \pi)$. The paper under review uses this approach to characterize tight wavelet frames associated with two symmetric or antisymmetric compactly supported refinable functions. All refinable masks of length up to 6 that satisfy this criterion are found.

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MSC:

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42C15 General harmonic expansions, frames

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