Zheng, Quan

**Coercive differential operators and fractionally integrated cosine functions.** (English)

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The author studies the integrated semigroups based on fractional integration. Let $X$ be a Banach space, $B(X)$ the space of linear bounded operators in $X$ and $\rho(B)$ denote the resolvent set of $B$. An operator $B \in B(X)$ is said to generate an $\alpha$-times integrated cosine function $C(t)$, $t \geq 0$, if there exists $\alpha \geq 0$ such that for large $\lambda \in \mathbb{R}$, $\lambda^2 \in \rho(B)$ and $\lambda 1 - \alpha(\lambda^2 I - B)^{-1}$ is the Laplace transform of $C(t)$, where $C : [0, \infty) \to B(X)$ is assumed to be an exponentially bounded and strongly continuous family. The main statement of the paper is the following result. Let a polynomial $P(\xi)$ be $r$-coercive (which means that $|P(\xi)|^{-1} = O(|\xi|^{-r})$ as $|\xi| \to \infty$) for some $r \in (0, m)$ and let $\omega = \sup_{\xi \in \mathbb{R}^n} P(\xi) < \infty$ . If $\overline{\rho(P(A))} \neq 0$, then $\overline{P}(A)$ generates a norm-continuous $\alpha$-times integrated cosine function $C(t)$, in case $\alpha > n\frac{2m-r}{2r}$. Besides this, $\|C(t)\| \leq M\left(1 + t^{\frac{n}{2}}\right) e^{\sqrt{\omega} t}$ when $\omega > 0$ with some power estimate depending on $\alpha$ when $\omega \leq 0$.

Reviewer: Stefan G. Samko (Faro)

**MSC:**

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|---|---|---|
| 47D09 | Operator sine and cosine functions and higher-order Cauchy problems | Cited in **4** Documents |
| 47F05 | General theory of partial differential operators | |

**Keywords:**

Coercive differential operators; functional calculus; integrated cosine function

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