Dawson, Donald A.; **Li, Zenghu**

Construction of immigration superprocesses with dependent spatial motion from one-dimensional excursions. (English) Zbl 1038.60082

Probab. Theory Relat. Fields 127, No. 1, 37-61 (2003).

It is shown that for each $\mu \in M(\mathbb{R})$ (finite Borel measures on $\mathbb{R}$) a unique probability measure Q_μ on $C([0,\infty), M(\mathbb{R}))$ exists such that, for each $\varphi \in C^2(\mathbb{R})$, under Q_μ,

$$M_t(\varphi) = \langle \varphi, w_t\rangle - \langle \varphi, \mu\rangle - \frac{1}{2}\rho(0)\int_0^t \langle \varphi'', w_s\rangle\, ds, \quad t \geq 0, \tag{1}$$

is a continuous martingale with quadratic variation process

$$\langle M(\varphi)\rangle_t = \int_0^t \langle \sigma\varphi^2, w_s\rangle\, ds + \int_0^t \int_{\mathbb{R}} \langle h(z-\cdot)\,\varphi', w_s\rangle^2\, dz\, ds \tag{2}$$

(w_s is the coordinate process), where

$$\rho(x) = \int_{\mathbb{R}} h(y-x)h(y)\, dy, \quad x \in \mathbb{R},$$

and h is a continuously differentiable function on $\mathbb{R}$ such that h and h' are square-integrable. The system $\{Q_\mu,\ \mu \in M(\mathbb{R})\}$ represents a superprocess with dependent spatial motion (SDSM), where $\rho(0)$ is the migration rate and σ is the branching rate. Whereas super-Brownian motion (which corresponds to independence of the spatial motion) starting from any $\mu \in M(\mathbb{R})$ immediately enters the space of absolutely continuous measures, SDSM starting from any μ immediately enters the space of atomic measures.

The objective of the paper is to construct a class of immigration diffusion processes related to the SDSM, which consists in replacing (1) by

$$M_t(\varphi) = \langle \varphi, w_t\rangle - \langle \varphi, \mu\rangle - \frac{1}{2}\rho(0)\int_0^t \langle \varphi'', w_s\rangle\, ds - \int_0^t \langle \varphi q(w_s,\cdot), m\rangle\, ds, \quad t \geq 0, \tag{3}$$

where m is a nontrivial σ-finite Borel measure on $\mathbb{R}$ and q is a Borel function on $M(\mathbb{R}) \times \mathbb{R}$ with certain regularity conditions. The martingale problem (1) and (3) represents an SDSM with interactive immigration determined by $q(w_s,\cdot)$ and reference measure m. A new method is required for the construction of this diffusion process. It is constructed as the pathwise unique solution of a stochastic integral equation carried by a stochastic flow and driven by a Poisson process of one-dimensional excursions. The stochastic integral equation provides information on the properties of the sample paths of the immigration diffusion, which are given in detail.

Reviewer: Louis G. Gorostiza (Mexico City)

MSC:

60J80	Branching processes (Galton-Watson, birth-and-death, etc.)
60G57	Random measures
60H20	Stochastic integral equations

Keywords:

Superprocess; Dependent spatial motion; Immigration; Excursion; Stochastic equation; Poisson random measure

Full Text: DOI arXiv