Miller, Sanford S.; **Mocanu, Petru T.**
Subordinants of differential superordinations. (English) Zbl 1039.30011
Complex Variables, Theory Appl. 48, No. 10, 815-826 (2003).

Let $\mathcal{H}$ be the class of functions analytic in U and $\mathcal{H}(a,n)$ be the subclass of $\mathcal{H}$ consisting of functions of the form $f(z)=a+a_nz^n+a_{n+1}z^{n+1}+\dots$. Let Ω and Δ be any sets in the complex plane $\mathbb{C}$, let $p\in\mathcal{H}$ and let $\phi(r,s,t;z):\mathbb{C}^3\times U\to\mathbb{C}$. In the present paper, the authors obtain conditions on Ω, Δ and ϕ for which the following implication holds: $\Omega\subset\{\phi(p(z),zp'(z),z^2p''(z);z)|z\in U\}\Rightarrow\Delta\subset p(U)$.

When Ω and Δ are simply connected domains with $\Omega,\Delta\neq\mathbb{C}$, the above implication becomes $h(z)\prec\phi(p(z),zp'(z),z^2p''(z);z)\Rightarrow q(z)\prec p(z)$, where h and q are the conformal mappings of U onto the domains Ω and Δ respectively. If p and $\phi(p(z),zp'(z),z^2p''(z);z)$ are univalent and if p satisfies the second order superordination $h(z)\prec\phi(p(z),zp'(z),z^2p''(z);z)$, p is the solution of the differential superordination. (If f is subordinate to F, then F is superordinate to f.) Subordinant and best subordinant are defined similarly like dominant and best dominant in case of differential subordination.

Denote by $\mathcal{Q}(a)$, the set of all functions $f(z)$, with $f(0)=a$, that are analytic and injective on $\overline{U}-E(f)$, where $E(f)=\{\zeta\in\partial U:\lim_{z\to\zeta}f(z)=\infty\}$, and are such that $f'(\zeta)\neq0$ for $\zeta\in\partial U-E(f)$. For a set Ω in $\mathbb{C}$ and $q\in\mathcal{H}(a,n)$ with $q'(z)\neq0$, the class of admissible functions $\Phi_n[\Omega,q]$ consists of those functions $\phi:\mathbb{C}^3\times\overline{U}\to\mathbb{C}$ that satisfy the admissibility condition: $\phi(r,s,t;\zeta)\in\Omega$, whenever $r=q(z)$, $s=zq'(z)/m$, $\operatorname{Re}(t/s)+1\leq(1/m)\operatorname{Re}[zq''(z)/q'(z)+1]$, where $\zeta\in\partial U$, $z\in U$ and $m\geq n\geq1$.

The principal result proved in the paper for second order differential superordinations is the following:

Theorem. Let h be analytic in U and $\phi:\mathbb{C}^3\times U\to\mathbb{C}$. Suppose that $\phi(q(z),zq'(z),z^2q''(z);z)=h(z)$ has a solution $q\in\mathcal{Q}(a)$. If $\phi\in\Phi_n[h(U),q]$, $p\in\mathcal{Q}(a)$ and $\phi(p(z),zp'(z),z^2p''(z);z)$ is univalent in U, then $h(z)\prec\phi(p(z),zp'(z),z^2p''(z);z)\Rightarrow q\prec p$ and q is the best subordinant.

By using the results for first order superordinations together with previously known results for differential subordinations, the authors have obtained several differential "sandwich theorems". Also a special second order differential superordination is considered. Some applications of the results of this paper was obtained recently by *T. Bulboaca* [Demonstr. Math. 35, No. 2, 287–292 (2002; Zbl 1010.30020)].

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MSC:

30C80	Maximum principle, Schwarz's lemma, Lindelöf principle, analogues and generalizations; subordination
30C45	Special classes of univalent and multivalent functions of one complex variable (starlike, convex, bounded rotation, etc.)
34A40	Differential inequalities involving functions of a single real variable
30C40	Kernel functions in one complex variable and applications

Keywords:

differential subordination; differential superordination; subordinant; univalent; convex; starlike

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