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On the local well-posedness of the Benjamin-Ono equation in $H^S(\mathbb{R})$. (English) Zbl 1039.35106

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The authors prove the following theorem for the Benjamin-Ono equation

$$u_t + \mathbf{H}u_{xx} + uu_x = 0, \quad u(0,x) = u_0(x), \tag{8}$$

where $\mathbf{H}$ denotes the Hilbert transform.

Fix $s > \frac{5}{4}$. Then for every $u_0 \in \mathbf{H}^s(\mathbb{R})$, there exist $T \geq ||u_0||_{\mathbf{H}^s}^{-4}$ and a unique solution of (8) on the time interval $[0,T]$ satisfying

$$u \in C([0,T], L^2(\mathbb{R})), \quad u_x \in L^1([0,T], L^\infty(\mathbb{R})).$$

Moreover, for any $R > 0$, there exists $T \geq R^{-4}$ such that the nonlinear map $u_0 \to u$ is continuous from the ball of radius R of $\mathbf{H}^s(\mathbb{R})$ to $C([0,T], \mathbf{H}^s(\mathbb{R}))$.

Conditions for an improvement of the theorem are given.

Reviewer: Thomas Ernst (Uppsala)

MSC:

35Q53 KdV equations (Korteweg-de Vries equations)
76B03 Existence, uniqueness, and regularity theory for incompressible inviscid fluids
42B25 Maximal functions, Littlewood-Paley theory

Cited in **86** Documents

Keywords:

well-posedness; Hilbert transform; Littlewood-Paley theory; Strichartz inequalities; Sobolev space

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