Yamato, Kazuo
The spectrum of the Milnor-Gromoll-Meyer sphere. (English) ⎣Zbl 1040.58012⎦
J. Math. Soc. Japan 54, No. 2, 409-445 (2002).

The author analyses the spectrum of the Milnor-Gromoll-Meyer sphere, i.e., a Riemannian manifold Σ^7 which is homeomorphic to the standard 7-sphere $\mathbb{S}^7$ but not diffeomorphic to $\mathbb{S}^7$. In particular, he shows that the eigenvalues $0 = \gamma_0 < \gamma_1 \leq \gamma_2 \leq \ldots$ of Σ^7 are uniformly close to an explicitly given sequence λ_l, i.e., there exists a positive constant c such that $|\gamma_l - \lambda_l| \leq c$ for all $l = 0, 1, 2, \ldots$. This result is not strong enough to "hear" the shape of Σ^7 but the author indicates that his calculations contribute to statistical properties of spectra.

The Milnor-Gromoll-Meyer sphere is defined as quotient $\Sigma^7 = \Gamma \setminus Sp(2)$ where $Sp(n)$ is the symplectic group for dimension n, i.e., the group of $n \times n$ quaternion matrices Q such that $QQ^* = Q^*Q = \mathrm{Id}$ with a normalized bi-invariant metric. Furthermore Γ is the action of the quaternions on $Sp(2)$ given by

$$\Gamma(q, Q) = \begin{pmatrix} q & 0 \\ 0 & q \end{pmatrix} Q \begin{pmatrix} \overline{q} & 0 \\ 0 & 1 \end{pmatrix},$$

[cf. *D. Gromoll* and *W. Meyer*, Ann. Math. (2) 100, 401–406 (1974; Zbl 0293.53015)].

More generally, the author calculates the spectrum of Riemannian manifolds M^7 given as quotient of $Sp(2)$ by certain actions of the quaternions. His examples comprise the exotic Milnor-Gromoll-Meyer 7-sphere as well as three other 7-spheres diffeomorphic to the standard sphere.

The main idea in the proof is to calculate the eigenspaces of the symplectic unitary group $SpU(4) = Sp(4, \mathbb{C}) \cap U(4)$, which is isomorphic to $Sp(2)$, and their subspaces of functions invariant under the actions.

Reviewer: Olaf Post (Aachen)

MSC:

58J50	Spectral problems; spectral geometry; scattering theory on manifolds	Cited in **1** Document
35P20	Asymptotic distributions of eigenvalues in context of PDEs	
43A75	Harmonic analysis on specific compact groups	
57R60	Homotopy spheres, Poincaré conjecture	

Keywords:

exotic sphere; spectral geometry; Milnor-Gromoll-Meyer sphere; Riemannian manifolds

Full Text: DOI