

Zudilin, V. V.

A third-order Apéry-like recursion for $\zeta(5)$. (English. Russian original) Zbl 1041.11057
Math. Notes 72, No. 5, 733-737 (2002); translation from *Mat. Zametki* 72, No. 5, 796-800 (2002).

The author constructs two hypergeometric sequences both taking values of the shape $A_n\zeta(5)+B_n\zeta(3)+C_n$ and thence a four-term (thus third order) linear difference equation with coefficients polynomials in the running index n and with solutions of the shapes $(q_n\zeta(5) - p_n)$ and $(q_n\zeta(3) - \tilde{p}_n)$. Here the sequences of rationals (p_n/q_n) , respectively \tilde{p}_n/q_n , converge to $\zeta(5)$, respectively $\zeta(3)$, at geometric rate, but not fast enough to prove irrationality – the q_n are integers, but the p_n and \tilde{p}_n have denominators the seventh power of the lcm $[1, 2, \dots, n]$.

Reviewer: Alf van der Poorten (Killara)

MSC:

11M06 $\zeta(s)$ and $L(s, \chi)$
11Y35 Analytic computations

Cited in **5** Documents

Keywords:

hypergeometric sequences; Riemann zeta-function

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