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The calculus of variations and functional analysis. With optimal control and applications in mechanics. (English) [Zbl 1042.49001](#)

Series on Stability, Vibration and Control of Systems, Series A 12. River Edge, NJ: World Scientific (ISBN 981-238-581-9/hbk). xiii, 420 p. (2003).

This is an interesting textbook devoted to problems of the classical calculus of variations, elements of modern optimal control, functional analysis and their applications to technology: mechanics, control, and computing. A unified and sensible treatment of these parts of mathematics, together with examples of how to exploit them in mechanics and in other fields of their applications, is the objective of this book. The book is written for readers with the standard mathematical background in engineering mathematics. Calculus of variations is presented in classical variant: the simplest problem and its various extensions, the first variation, Euler's equations, extremals with movable ends, Weierstrass-Erdmann conditions, isoperimetric problems, sufficient conditions for minimum. The exposition is a simple, continued exploitation of the same technique to prevent real progress and to introduce the ideas applicable to more complex problems. Introduction to optimal control problems is given by detailed investigation of the terminal control problem and its generalization under geometrical restrictions on control actions, Pontryagin's maximum principle is proved in the most simplest way by using the notion of needle variation and properties of solutions of ordinary differential equations. Optimal time control problems are discussed without proofs. Some problems of mechanics and physics are studied from the viewpoint of the calculus of variations. Both equilibrium and dynamic problems are investigated with the aid of the virtual work and the Hamilton-Ostrogradskij principles. The exposition is based on tools of functional analysis presented previously for this purpose. Generalized setup of problems, existence and uniqueness of solutions are discussed. Approximation methods of Ritz, Bubnov-Galerkin, Fourier and their convergence properties are investigated. Throughout the book the authors explain the background behind the various methods in use: how they arrive at approximations, in what situations they are applicable, and how much accuracy they can provide. The reader will be able to understand modern work in this important area presented.

From the foreword by Ardeshir Guran: "It is lucid, well-connected, and concise. The material has been carefully chosen. Throughout the book, the authors lay stress on central ideas as they present one powerful mathematical tool after another. The reader is thus prepared not only to apply the material to his or her own work, but also to delve further into the literature if desired. The book could function as both (i) an attractive textbook for a course on engineering mathematics at the graduate level, and (ii) a useful reference for researchers in mechanics, electrical engineering, computer science, mechatronics, or related fields such as mechanical, civil, or aerospace engineering, physics, etc. It may also appeal to those mathematicians who lean toward applications in their work. The presence of homework problems at the end of each chapter (and hints for selected exercises at the end of the book) will facilitate its use as a textbook."

Thus the book is a significant contribution to the process of successful preparation of engineering students. It is also a good methodical foundation for engineers and lectures in their work.

Reviewer: [Ernst G. Al'brekht \(Ekaterinburg\)](#)

MSC:

- [49-01](#) Introductory exposition (textbooks, tutorial papers, etc.) pertaining to calculus of variations and optimal control
- [47-01](#) Introductory exposition (textbooks, tutorial papers, etc.) pertaining to operator theory
- [70Q05](#) Control of mechanical systems
- [74M05](#) Control, switches and devices ("smart materials") in solid mechanics
- [00A06](#) Mathematics for nonmathematicians (engineering, social sciences, etc.)

Cited in **5** Documents

Keywords:

[textbook](#); [calculus of variations](#); [optimal control](#)