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Projective transformations of pseudo-Riemannian manifolds. (English) Zbl 1043.53054

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This paper is a survey of the results on the theory of projective transformations of pseudo-Riemannian manifolds. A projective transformation of a pseudo-Riemannian manifold M^n is an automorphism of the induced Riemannian connection of the projective structure that takes geodesics of M^n into geodesics. The theory of projective transformations on spaces endowed with a linear connection was developed by É. Cartan, I. P. Egorov, L. P. Eisenhart, G. Fubini, M. S. Knebelman, S. Kobayashi, S. Lie, I. Mikesh, I. A. Schouten, P. A. Shirokov, N. S. Sinyukov, A. S. Solodovnikov, T. Y. Thomas, G. G. Vranceanu, and others. In particular, A. S. Solodovnikov, following G. Fubini's papers, found a classification of the Riemannian spaces $M^n, n \geq 3$, in terms of local groups of projective transformations.

The present author [Tr. Geom. Semin. 6, 295-316 (1974; Zbl 0309.53046)] proposed a new approach for solution of this problem when the metric is not positive definite. She determined all pseudo-Riemannian manifolds with Lorentz signature $(+, -, \ldots, -)$ (Lorentz manifolds) of dimension $n \geq 3$ admitting nonhomothetic infinitesimal projective and affine transformations, and for each of them the maximal projective and affine Lie algebras along with the homothetic and isometric subalgebras were found. The author finds the solution of the classical geometrical problem of determining the pseudo-Riemannian metrics with corresponding geodesics and the Lie problem. By using the technique of skew-normal frames [Sov. Math. 26, No. 6, 76–81 (1982; Zbl 0509.53019)] she determines all 2-dimensional pseudo-Riemannian manifolds that admit Lie algebras of infinitesimal projective transformations. Special attention is given to concircular geometry, which is closely related to projective transformations, as well to applications in physics, mechanics, and the theory of differential equations.

Finally, the author studies the projective geometry of differential systems. She gives a classification of second-order differential equations whose right-hand sides are third-degree polynomials in the derivatives of the unknown function and systems of second-order differential equations in two unknowns and quadratic right-hand sides (geodesic equations) with respect to the Lie algebras of infinitesimal symmetries. Also, she finds conditions under which the integral curves of the above-mentioned equations are straight lines. The paper contains pictures of a number of revolution surfaces admitting projective transformations. The bibliography contains 422 items.

Reviewer: Andrew Bucki (Oklahoma City)

MSC:

53C50	Global differential geometry of Lorentz manifolds, manifolds with indefinite metrics
34A26	Geometric methods in ordinary differential equations
53B30	Local differential geometry of Lorentz metrics, indefinite metrics
34C14	Symmetries, invariants of ordinary differential equations
53B10	Projective connections

Keywords:

projective transformation; infinitesimal projective transformation; concircular geometry

Full Text: DOI