Zudilin, V. V.

Diophantine problems for q-zeta values. (English. Russian original) Zbl 1044.11066

Math. Notes 72, No. 6, 858-862 (2002); translation from Mat. Zametki 72, No. 6, 936-940 (2002).

Consider the function of two variables

$$\zeta_q(k) = \sum_{n=1}^{\infty} n^{k-1} \frac{q^n}{1-q^n},$$

for $|q| < 1$, $k = 1, 2, \ldots$, which is a q-analogue of the Riemann zeta function in the following sense:

$$\lim_{q \to 1} (1-q)^k \zeta_q(k) = (k-1)! \zeta(k).$$

The author first presents some of his results about the diophantine results of $\zeta_q(k)$ when $1/q \in \mathbb{Z} \setminus \{\pm 1\}$: in particular, he outlines how he adapted the Rhin & Viola's group method in the q-hypergeometric setting and how it enabled him to get remarkable new irrationality measures for $\zeta_q(1)$ and $\zeta_q(2)$.

He concludes with some remarks about the nature of $\zeta_q(k)$ as a function of q: when $k \geq 4$ is even, the functions $\zeta_q(k)$ are modular forms of weight k, hence they belong to $\mathbb{Q}[\zeta_q(2), \zeta_q(4), \zeta_q(6)]$ (which can be viewed as a functional q-analogue of $\zeta(k) \in \mathbb{Q} \cdot \pi^k$ for even $k \geq 2$), whereas nothing such is known for odd $k \geq 1$.

Since it is conjectured that π and the numbers $\zeta(2j+1)$ $(j \geq 1)$ are algebraically independent over $\mathbb{Q}$, the author proposes the following q-counterpart: prove that the functions $\zeta_q(2), \zeta_q(4), \zeta_q(6)$ and $\zeta_q(2j+1)$ $(j \geq 0)$ are algebraically independent over the field $\mathbb{C}(q)$.

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MSC:

11J82 Measures of irrationality and of transcendence
11J72 Irrationality; linear independence over a field

Keywords:

irrationality measures

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