Zudilin, Wadim

Remarks on irrationality of q-harmonic series. (English) Zbl 1044.11068

Manuscr. Math. 107, No. 4, 463-477 (2002).

Given some $p \in \mathbb{Z} \setminus \{0, \pm 1\}$, we may consider the two series

$$h_p(1) = \sum_{\nu=1}^{\infty} \frac{1}{p^\nu - 1} \quad \text{and} \quad \ln_p(2) = \sum_{\nu=1}^{\infty} \frac{(-1)^{\nu-1}}{p^\nu - 1}.$$

The author improves the best currently known upper bound for the irrationality measure μ of $h_p(1)$ and $\ln_p(2)$. He proves that for all $p \in \mathbb{Z} \setminus \{0, \pm 1\}$:

$$\mu(h_p(1)) \leq 2.49846482... \quad \text{and} \quad \mu(\ln_p(2)) \leq 3.29727451...$$

To prove these results, he first introduces a basic hypergeometric series with a particular form which enables him to construct rational linear forms in 1 and $h_p(1)$, resp. 1 and $\ln_p(2)$. He then adapts the (by now well-known) methods used to deal with ordinary hypergeometric series. In particular, he adapts a classical process used to extract "big" common factors from the coefficients of the rational linear forms .

Reviewer: Tanguy Rivoal (Caen)

MSC:

11J82	Measures of irrationality and of transcendence
33D15	Basic hypergeometric functions in one variable, ${}_r\phi_s$

Full Text: DOI