

Kisin, Mark

Overconvergent modular forms and the Fontaine-Mazur conjecture. (English) Zbl 1045.11029
Invent. Math. 153, No. 2, 373-454 (2003).

Let $\rho : G_{\mathbb{Q}} \rightarrow \mathrm{GL}_m(\mathbb{Q}_p)$ be a continuous irreducible representation of $G_{\mathbb{Q}} = \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, which is unramified outside finitely many primes and whose restriction to a decomposition group at p is potentially semistable. *J.-M. Fontaine* and *B. Mazur* [Elliptic curves, modular forms, and Fermat's last theorem, Ser. Number Theory 1, 41–78 (1995; [Zbl 0839.14011](#))] made the following conjectures:

(i) ρ appears (up to a twist) as a subquotient in the cohomology of a finite type \mathbb{Q} -scheme (i.e. ρ is “geometric”),

(ii) Let E/\mathbb{Q}_p be a finite extension, and let $\rho : G_{\mathbb{Q}} \rightarrow \mathrm{GL}_2(E)$ be a continuous, odd, irreducible representation, unramified outside finitely many primes, whose restriction to a decomposition group at p is potentially semistable when regarded as a \mathbb{Q}_p -representation. Then, up to a twist, ρ arises from a modular form.

The author proves that, apart from a certain exceptional case, the Fontaine-Mazur conjecture is true for representations coming from finite slope, overconvergent eigenforms of weight $k \neq 1$ (Theorem 6.6). One of the key ingredients in the proof is the theory of the “eigencurve” of *R. Coleman* and *B. Mazur* [Galois representations in arithmetic algebraic geometry, Lond. Math. Soc. Lect. Note Ser. 254, 1–113 (1998; [Zbl 0932.11030](#))]. As an application of this theorem (more precisely, the technical result Theorem 6.3), he answers a question of *F. Q. Gouvêa* [Arithmetic of p -adic modular forms, Lectures Notes in Mathematics 1304, Springer-Verlag, Berlin (1988; [Zbl 0641.10024](#))], who asked to what extent the u_p -eigenvalue of an overconvergent modular form f is determined by its Fourier coefficients $a_n(f)$ with pX_n . As a second application of Theorem 6.3 he recovers the result of *B. Mazur* and *A. Wiles* [Compos. Math. 59, 231–264 (1986; [Zbl 0654.12008](#))], which says that representations attached to ordinary p -adic modular forms are ordinary.

Reviewer: [A. Dabrowski \(Szczecin\)](#)

MSC:

[11F33](#) Congruences for modular and p -adic modular forms
[11F80](#) Galois representations
[11F30](#) Fourier coefficients of automorphic forms

Cited in **4** Reviews
Cited in **51** Documents

Keywords:

overconvergent modular form; Fontaine-Mazur conjecture; Galois representations; p -adic modular form; deformation theory

Full Text: [DOI](#)