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A basic introduction to surgery theory. (English) Zbl 1045.57020

Farrell, F. Thomas (ed.) et al., Topology of high-dimensional manifolds. Proceedings of the school on high-dimensional manifold topology, Abdus Salam ICTP, Trieste, Italy, May 21–June 8, 2001. Number 1 and 2. Trieste: The Abdus Salam International Centre for Theoretical Physics (ISBN 92-95003-12-8/pbk). ICTP Lect. Notes 9, 1-224 (2002).

This is a very well written introductory book on surgery theory (of not necessarily simple connected manifolds) and its applications to the classification of manifolds. The first six chapters offer a very clear presentation of "classical" material (basic results contained in well-known books of Browder and Wall and papers by Milnor and Keirvaire on groups of homotopy spheres). In many places the presentation makes use of some further developments, mainly due to Ranicki, and is considerably more readable than the classical monograph of Wall and requires only basic knowledge (of good quality) of algebraic and differential topology. Moreover, the book is reasonably self-contained (in particular basic material on handle decompositions and Whitehead torsion is included as well as some necessary theorems due to Haefliger, Hirsch, Smale and Whitney on imbeddings and immersions), thus it could be a good starting point for the study in this branch of mathematics. The last two chapters, more advanced and much less self-contained, provide a brief discussion of some recent developments related to assembly maps and conjectures of Farrell-Jones and Baum-Connes and relationships between these conjectures and the Novikov conjecture (homotopy invariance of higher signatures), the Borel conjecture (any homotopy equivalence between two closed aspherical manifolds is homotopic to a homeomorphism) and the stable Gromov-Lawson-Rosenberg conjectures on existence of metrics with positive scalar curvature.

Now a little more detailed review of the book follows. The first chapter intended to be a justification of and introduction to the surgery method provides a proof of the s-cobordism theorem. The chapter starts with some basic information on handle decompositions and their relations to CW-complexes, as well as a definition of the Whitehead group (using elementary operations on matrices).

Chapter 2 offers a reasonably self-contained review of the Whitehead torsion. Both, the algebraic and geometric theory is presented as well as the relationship between the two (homotopy equivalence is simple if and only if its Whitehead torsion vanishes). Concise proofs of basic theorems are given (the Chapman theorem on the topological invariance of Whitehead torsion is an exception). As an application of theories developed in the first two chapters the author presents the homotopy and diffeomorphism classification of lens spaces with almost full proofs of the main theorems.

In Chapter 3 principal notions of the surgery theory are briefly discussed. Namely the author defines normal maps, normal invariants and normal bordisms and gives a concise account of the theory of Poincaré duality (in the non-simply connected and non-orientable setting) and Poincaré complexes as well as the Spivak fibration and Thom invariants. Some technical homotopy-theoretic proofs of basic theorems on spherical fibrations are only sketched (e.g., existence of the Spivak fibration), some others (e.g., uniqueness of the Spivak fibration) are omitted here. The chapter is concluded by a brief discussion of framed surgery on normal maps (after recalling basic facts on immersions and regular homotopies).

Chapter 4 offers a brief review of the algebraic theory of Wall surgery obstruction groups. First the author presents the theory of intersections and self-intersections of immersed submanifolds and their relation to cap products and homology pairings as needed in the non-simply connected setting. Next, the theory of quadratic forms over group algebras and surgery kernels is discussed, and the basic theorem of surgery theory for even dimensional manifolds (if the quadratic form on the surgery kernel is stably hyperbolic the normal map is normally bordant to a (simple) homotopy equivalence) is proved. Finally, the definition of Wall surgery L-groups and a sketch of the proof of the main surgery obstruction theorem (independence of the surgery obstruction on the normal map within given normal bordism class and vanishing of the surgery obstruction as necessary and sufficient conditions for a normal map to be normally bordant to a homotopy equivalence) are given. After discussing the even-dimensional case the author turns to the (more complicated) odd-dimensional case, theory of formations, odd-dimensional surgery obstructions and odd-dimensional Wall surgery groups. The presentation of the material in the odd-dimensional case is less complete than that for even dimensions due to the more complicated and technical nature of the matter. By now the author dealt with closed manifolds and homotopy equivalences. The fourth chapter is concluded by a presentation of the variant of surgery theory for manifolds with not necessarily empty boundary and in the "simple homotopy equivalence" setting.

The fifth chapter is devoted to the presentation of the surgery exact sequence, which is the main tool for studying manifolds. In this chapter a theorem on realizability of surgery obstructions (any element of the Wall surgery group is the surgery obstruction for a surgery problem) as well as PL and TOP variants of principal theorems of surgery theory are briefly discussed.

The sixth chapter offers a detailed presentation of classical results, mainly due to Kervaire and Milnor, on groups of homotopy spheres (including Milnor's exotic 7-spheres, the long surgery sequence for homotopy spheres, J-homomorphism, Kervaire-Milnor braid etc.).

The final two chapters are intended to facilitate access to recent papers in surgery theory and some related topics of the geometry of manifolds as mentioned above.

For the entire collection see [Zbl 0996.00038].

Reviewer: Wieslaw Oledzki (Bialystok)

MSC:

57R67 Surgery obstructions, Wall groups

57-02 Research exposition (monographs, survey articles) pertaining to manifolds and cell complexes

57R65 Surgery and handlebodies

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surgery; L-theory; Whitehead torsion; s-cobordism; Baum-Connes conjecture