Beg, Ismat

Fixed points of fuzzy monotone maps. (English) Zbl 1047.03044

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This short paper presents two theorems proving sufficient conditions for the existence of fixed points of a mapping that is fuzzy monotone with respect to a fuzzy order. From the various existing definitions of a fuzzy order, the author uses the following one, relying mainly on the ideas proposed in 1986 by C. Z. Luo and in 1987 by A. Billot [see, e.g., *A. Billot*, Economic theory of fuzzy equilibria. An axiomatic analysis. 2nd ed. (Springer, Berlin) (1995; Zbl 0861.90017)]: A fuzzy order on a crisp set X is a normalized fuzzy binary relation R on X with a membership function r fulfilling the conditions $(\forall x, y \in X) r(x,y) \star r(y,x) > 0 \Rightarrow x = y$ (antisymmetry), and $(\forall x, y, z \in X) r(x,y) \geq r(y,x) \& r(y,z) \geq r(z,y) \Rightarrow r(x,z) \geq r(z,x)$ (f-transitivity), where $\star$ denotes the Łukasiewicz t-norm. In both theorems, a basic part of the sufficient condition is the requirement $(\exists x \in X) r(x, f(x)) \geq r(f(x), x)$. In addition, the first theorem requires every fuzzy chain in X to have a supremum, whereas the second one requires only every countable fuzzy chain in $\{y : y \in X \& r(x,y) \geq r(y,x)\}$ to have a supremum; instead the mapping is required to be not only fuzzy monotone, but also fuzzy order-continuous. A substantial difference between both theorems consists in the way how the existence of a fixed point of the considered mapping f is established: In the first theorem, its existence follows from Fuzzy Zorn's Lemma, whereas in the second theorem it is $\sup_n f^n(x)$ that is proved to be a fixed point of f, and the existence of that supremum is already assumed in the sufficient condition.

The theorems are proved in the setting of fuzzy sets with $[0,1]$-valued membership functions. In the reviewer's opinion, they can be proved also with the more general membership functions assuming values in a linearly ordered lattice, and also the t-norm in the definition of a fuzzy order can be quite general.

Reviewer: Martin Holeňa (Praha)

MSC:

03E72 Theory of fuzzy sets, etc.
06A06 Partial orders, general

Keywords:

fuzzy order; fuzzy chain; fuzzy Zorn's lemma; fuzzy monotone mapping; fuzzy order-continuous mapping; fixed points of fuzzy mappings

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